

## Elasticity of substitution

$$\sigma = \frac{\omega dk}{k d\omega}$$

$\sigma = 1$       Cobb-Douglas

$\sigma = 0$       Leontief, perfect complements

$\sigma = \infty$       Straight line isoquant, perfect substitutes

- Ken-ichi Inada

$$f(0) = 0$$

$$f(\infty) = \infty$$

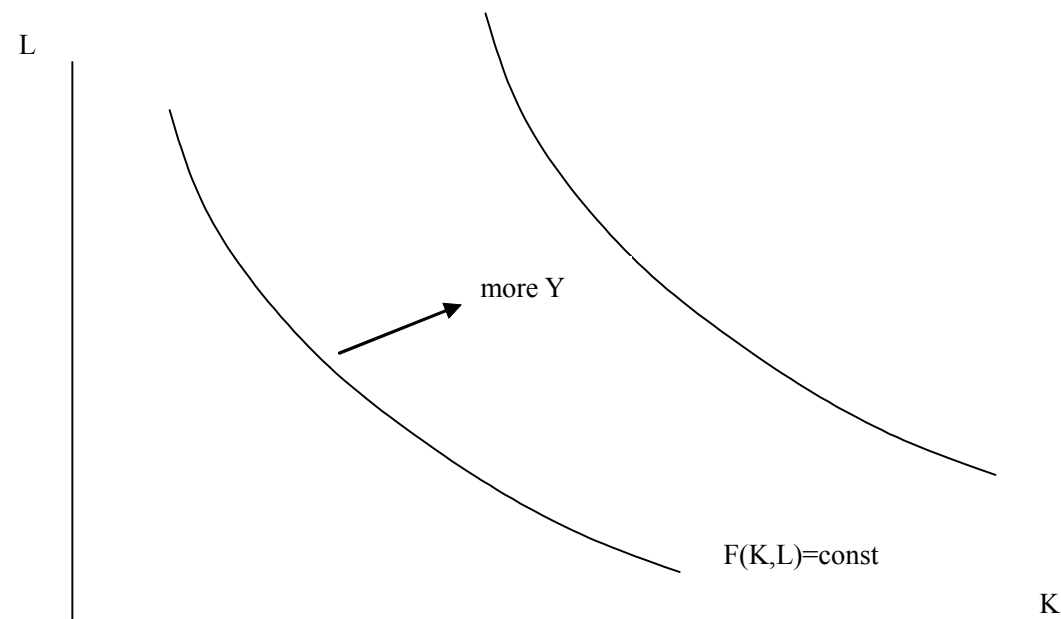
$$f'(0) = \infty$$

$$f'(\infty) = 0$$

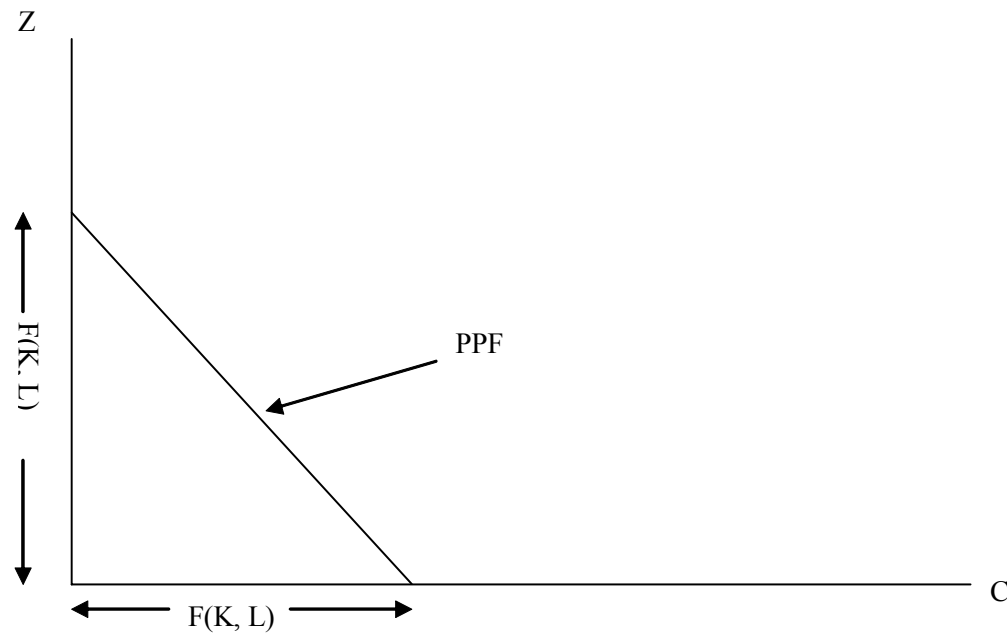
Satisfied by CD, but not by all interesting concave PF.

## Table Top and Isoquants

- 3 dimensional production function in 3 dimentions
- 3 dimensional produciton function in 2 dimentions



## Flat PPF



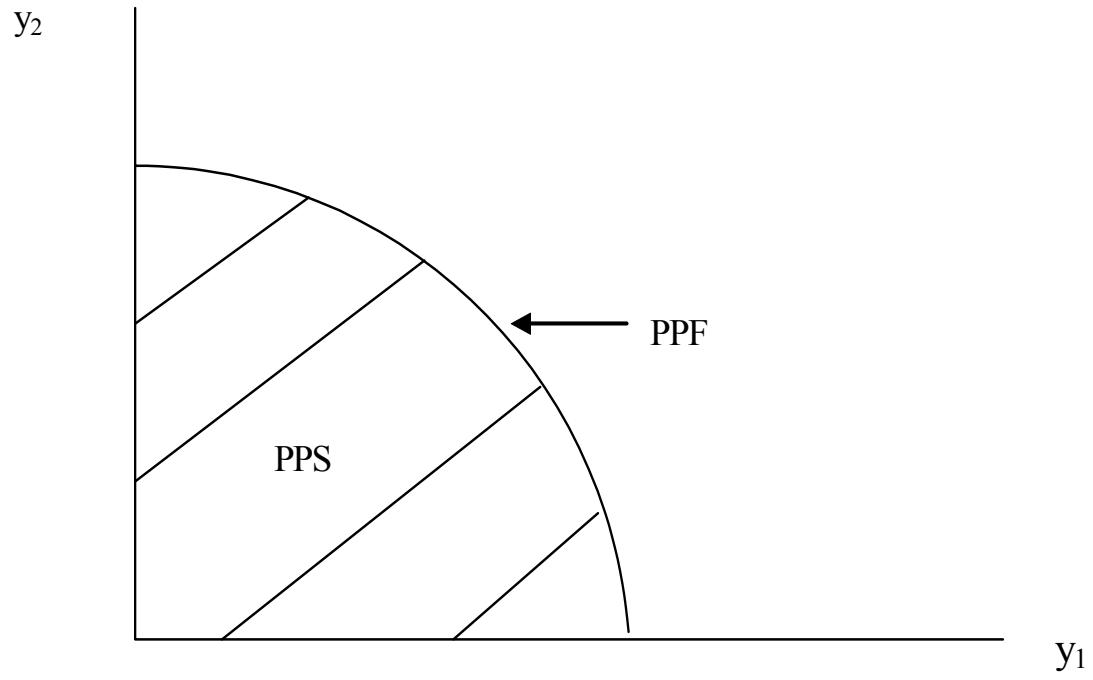
PPF is flat. Here with slope =  $-1$

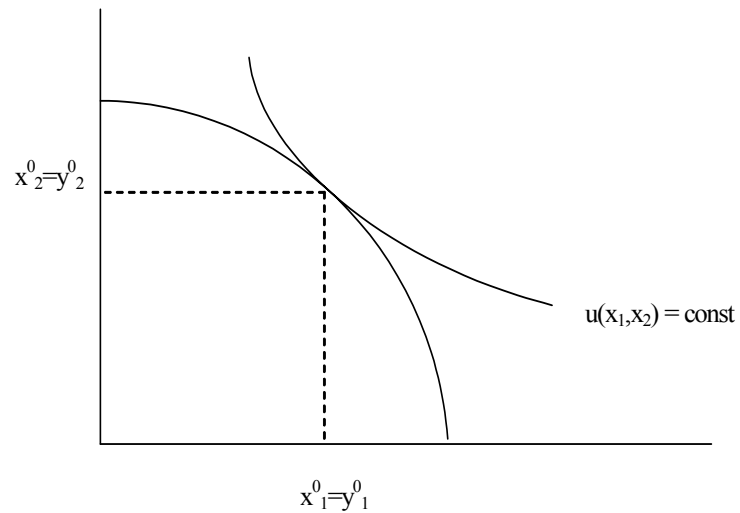
Non specialization price is fixed by production.

General Equilibrium:

Profit max is not merely an assumption. Should be a result, when applicable.

Examples of where not applicable.





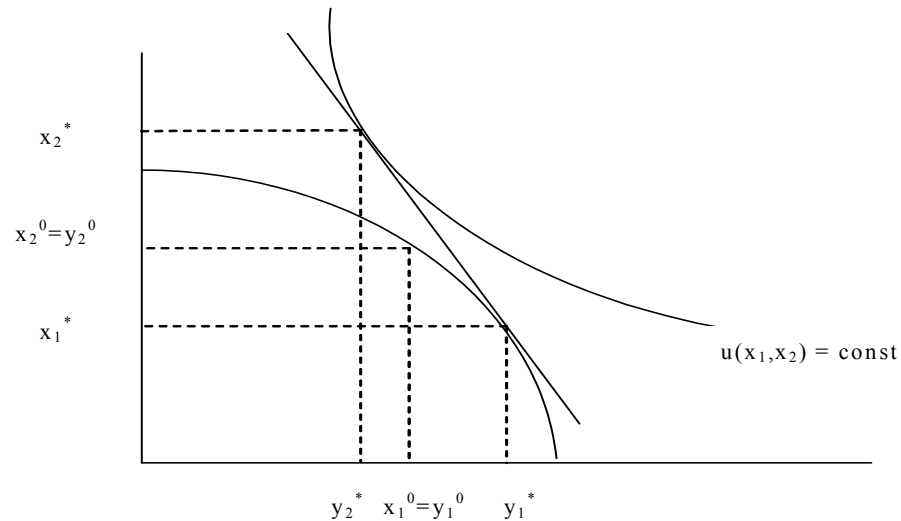
Robinson Crusoe

$$\max u(x_1, x_2)$$

subject to  $(x_1, x_2) = (y_1, y_2)$

and  $(y_1, y_2) \in PPS$

Solution:  $(x_1^0, x_2^0) = (y_1^0, y_2^0)$



## Trade under Perfect Markets

$$\begin{aligned} & \max u(x_1, x_2) \\ \text{subject to} & \quad p_1 x_1 + p_2 x_2 = p_1 y_1 + p_2 y_2 \\ \text{and} & \quad (y_1, y_2) \in PPS \end{aligned}$$

Solution:  $(x_1^*, x_2^*, y_1^*, y_2^*)$

Typically  $x_1^* \neq y_1^*$  and  $x_2^* \neq y_2^*$

Autarky (only special case)  $x_1^* = y_1^*$  and  $x_2^* = y_2^*$

Profit maximization is a theorem under perfect competition.

$$\max \pi = p_1 y_1 + p_2 y_2$$

is necessary for constrained utility maximization.

## Intertemporal Analogue

$$(x^t, x^{t+1}), (y^t, y^{t+1}), p^t, p^{t+1}$$

Define perfect competition interest rate  $r^t$  by

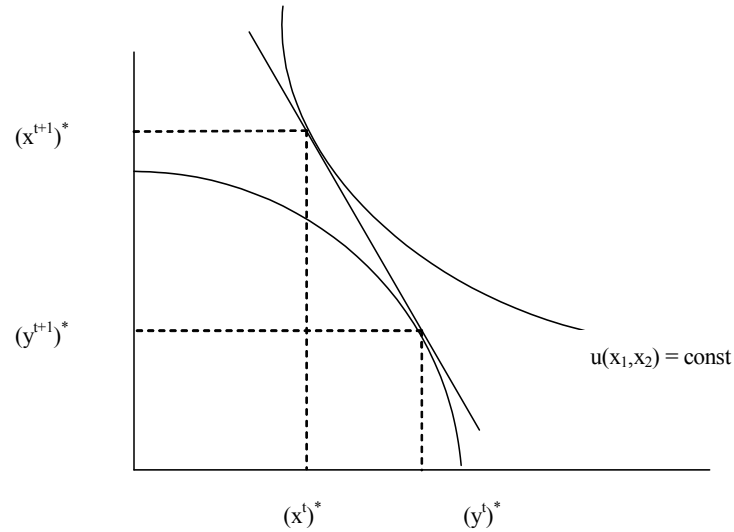
$$\frac{p^{t+1}}{p^t} = \frac{1}{1 + r^t}$$

and interest factor  $R^t$  by

$$R^t = (1 + r^t)$$

$p^{t+1}/p^t$  is the price today of chocolate delivered tomorrow.

## Irving Fisher



$$\max u(x^t, x^{t+1})$$

$$\text{subject to } x^t + \frac{x^{t+1}}{1+r^t} = y^t + \frac{y^{t+1}}{1+r^t}$$

$$\text{and } (y^t, y^{t+1}) \in PPS$$

maximizing PV (or PDV) is necessary for maximizing  $u(x^t, x^{t+1})$ .

## Summary

1. Robinson Crusoe produces to his own tastes. He produces at the point on the PPF that maximizes his utility.
2. If trade is possible and perfectly competitive, then one produces to the market as coded in the price ratio  $(p_2/p_1)$  in order to maximize utility. It is a result that the individual desires his firm to maximize profits.
3. The intertemporal analogue is that the individual desires his firm to maximize PV.

## Uncertainty

Arrow-Debreu contingent claims.

2 states of nature:

$s = \alpha$  is rain

$s = \beta$  is drought

$x_\alpha$  delivery of  $x_\alpha$  bushels of wheat if rain, 0 otherwise.

$x_\beta$  delivery of  $x_\beta$  bushels of wheat if rain, 0 otherwise.

$p_\alpha$  and  $p_\beta$  are prices of contingent claims

$$\text{VNM: } V(x_\alpha, x_\beta) = \pi(\alpha) u(x_\alpha) + (1 - \pi(\alpha)) u(x_\beta)$$

$$\text{subject to } p_\alpha x_\alpha + p_\beta x_\beta = p_\alpha y_\alpha + p_\beta y_\beta$$

$$\text{and } (y_\alpha, y_\beta) \in PPS$$

Maximize profits at contingent claims prices. **Not** maximize "expected" profits.

Arrow-Debreu contingent claims

Arrow-money or Arrow securities and spot markets

Irving Fisher analogue:

Forward market

money market and spot markets

