

Economics 613: Macroeconomics I

Fall 2006

Cornell University

Problem Set #11

Due: Monday, November 27, 2006

OG Model with Production

Consider an overlapping generations model in which each person lives for two periods. People work in the first period, when they are young, retire in the second period, when they are old, and then die off. People consume in both periods of life, and they pay for consumption in the second period by saving in the first period.

We shall refer to the cohort that is born at time t as generation t . Members of this generation are young in period t and old in period $t + 1$. Therefore, during period t , the young of generation t overlap with the old of generation $t - 1$. The population growth rate is n and there is no capital depreciation. Each person is endowed with 1 unit of labor and he does not derive utility from leisure.

Preferences are represented by:

$$U_t = \frac{c_{y,t}^{1-\theta} - 1}{1-\theta} + \beta \frac{c_{o,t}^{1-\theta} - 1}{1-\theta},$$

where $\theta > 0$ and $0 < \beta < 1$. Note that when $\theta \rightarrow 1$, then $\frac{c^{1-\theta}-1}{1-\theta} \rightarrow \log c$.

Technology:

$$Y_t = AK_t^\alpha L_t^{1-\alpha},$$

where $A > 0$, and $0 < \alpha < 1$.

(a) Consider an individual born at time t . Let s_t be the amount he saved in period t . From his lifetime utility maximization problem, get s_t as a function of wage income w_t and the interest rate r_{t+1} .

(b) Let $k_t = K_t/L_t$. Get the difference equation of k in the competitive equilibrium.

(c) Suppose $\theta = 1$. Then $U_t = \log c_{y,t} + \beta \log c_{o,t}$. Compute the steady state capital labor ratio k^* and analyze the full dynamics.