Economics 613: Macroeconomics I
Fall 2006
Cornell University
Problem Set #9
Due: Wednesday, November 1

1 Solow Growth Model, Continuous Time

1. Impulse diagrams. Suppose that the economy is in a steady-state, but at time t_0 the savings rate jumps from s_b to s_a where $s_a < s_b$ (as given in the diagram). Complete the graphs on the following page.

2. Describe how the following affects the continuous-time Solow diagram and the steady state.

(a) The depreciation rate falls.
(b) The production function is Cobb-Douglas, and capital's share rises.
(c) Workers exert more effort, so it takes 50 minutes to do what it formerly took 60 minutes to do.

2 Leontief Production Function

$$C + Z = Y = \min[aK, bL]$$

(a) Write down the intensive production function and plot y on the vertical axis versus k on the horizontal axis.

(b) Using this production function, study the full dynamics of (i) the Solow model in continuous time, (ii) the Ramsey-Cass-Koopmans model in continuous time.

3 Optimal Growth

For the following optimal growth problem

$$\max \int_0^\infty U(c(t))e^{-\delta t} dt$$

$$\begin{aligned} s.t.\ c(t) &= f(k(t)) - \lambda k(t) - \dot{k}(t) \\ k(0) &= k_0,\ \delta > 0 \\ U'(c) &> 0, U''(c) < 0,\ for\ c > 0 \\ U'(0) &= +\infty, .f(k) > 0,\ f'(k) > 0,\ f''(k) < 0\ for\ k > 0, \end{aligned}$$

show that the path satisfying the Euler equation (or the Hamiltonian equations) and tending to the rest point (q^*, k^*) is the optimal path.

4 Optimal Growth, Cont.'d.

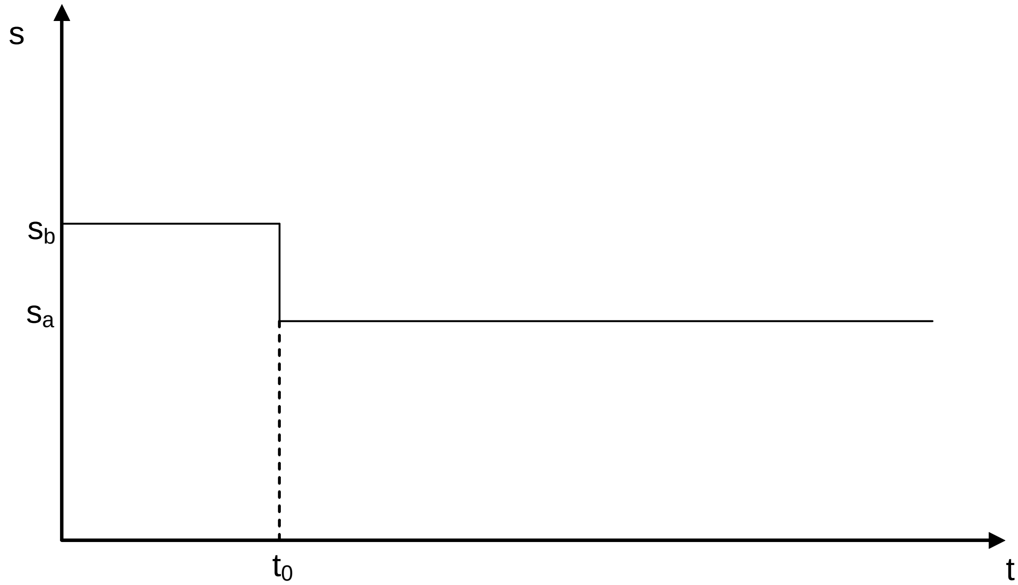
For problem 3, show that the Euler equations are equivalent to (or closely related to) the corresponding Hamiltonian equations from Pontryagin's Maximum Principle.

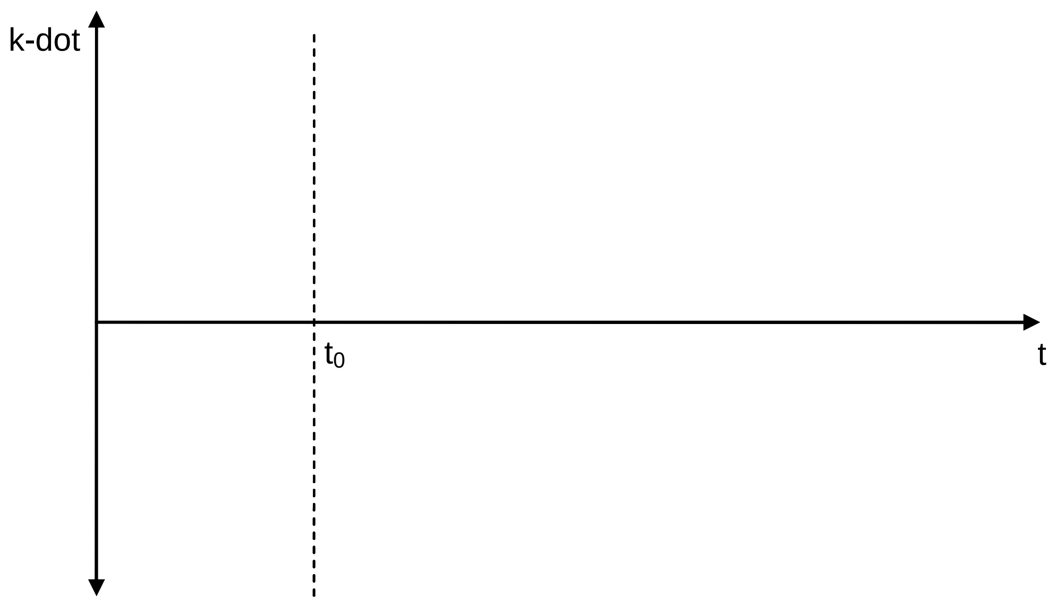
5 Ramsey-Cass-Koopmans Model

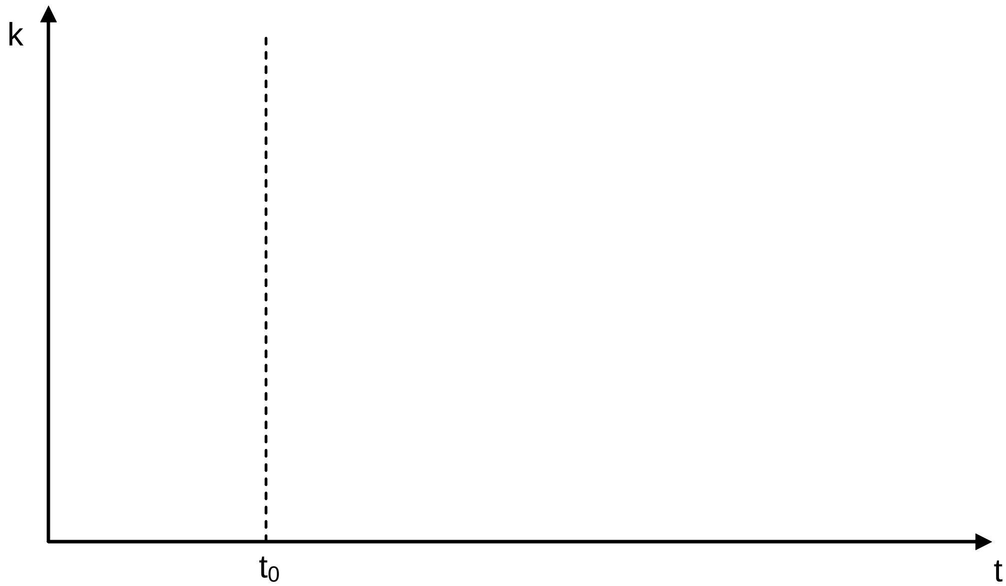
Suppose the economy is at balanced growth, but that at time t_0 a tax rate τ on capital income is unexpectedly instituted. Assume that the proceeds of the tax are distributed to people as lump-sum transfers.

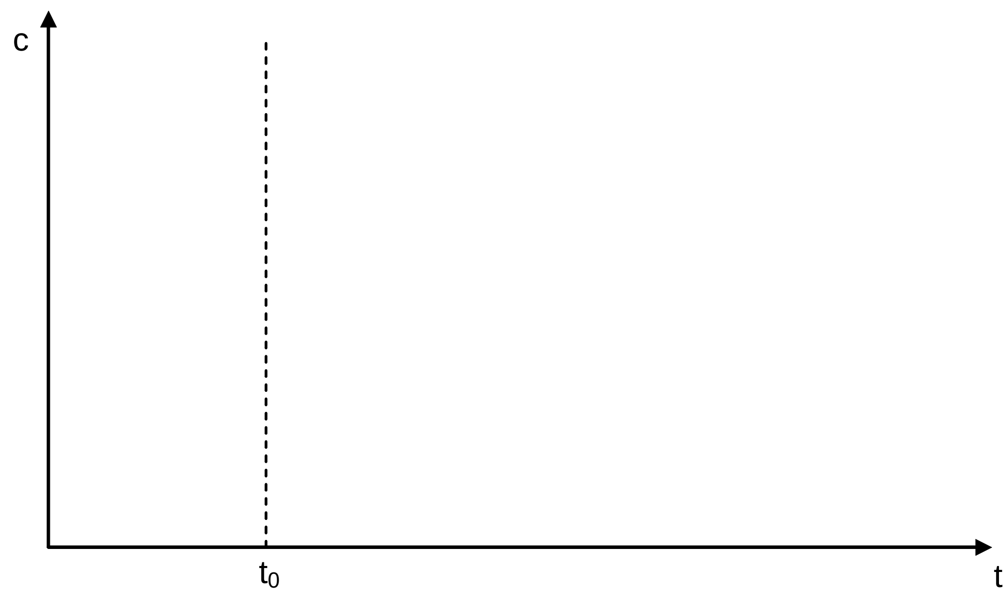
(a) What is the after-tax rate of return to the household?
(b) How does the tax rate affect the $\dot{q} = 0$ locus? The $\dot{k} = 0$ locus?
(c) How does the new balanced growth path compare to the old?

Diagrams in Problem Set #9






k
t_0
t


c
t_0
t