

Professor Karl Shell  
March 09, 2005

**Economics 614: Macroeconomics II**

Spring 2005

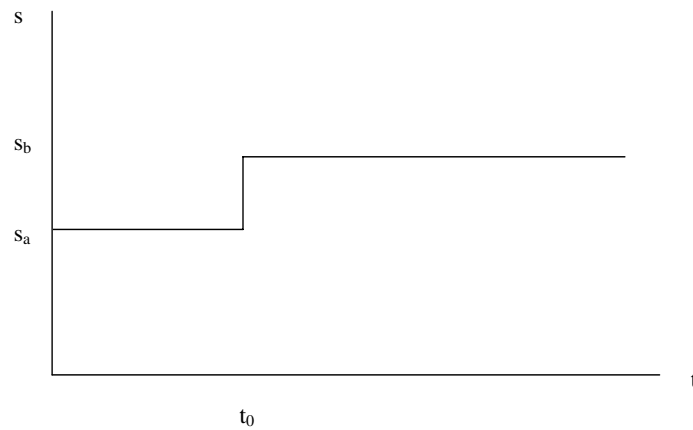
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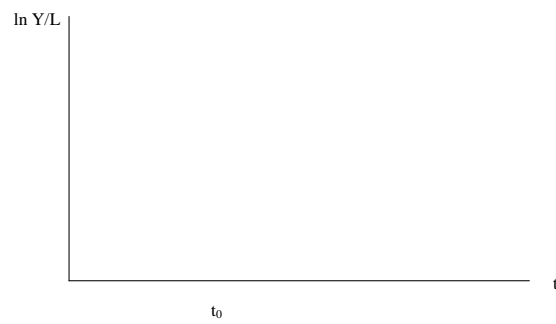
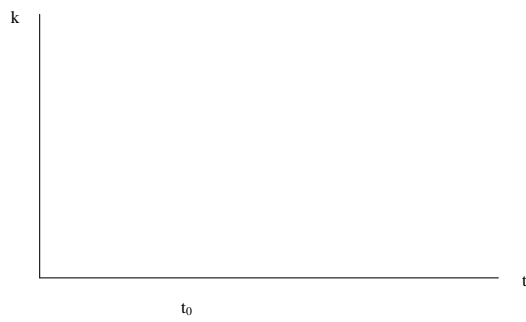
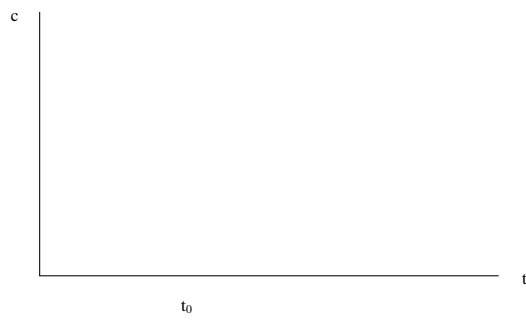
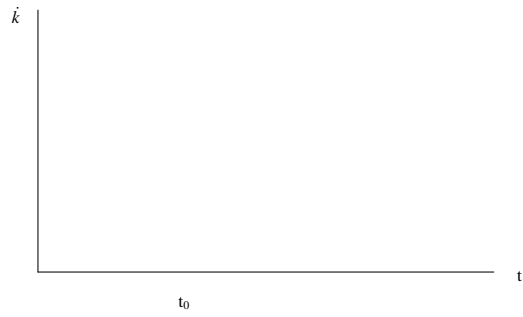
**Problem Set #7**

Due: Friday, March 18, 2005

**1 Solow Growth Model, Continuous Time.**

Impulse diagrams. Suppose the economy is in a steady-state, but at time  $t_0$  the savings rate jumps from  $s_a$  to  $s_b$ , where  $s_b > s_a$  (as below). Complete the graphs on the following page.





## 2 Question 1 Continued

Describe how the following affects the Solow diagram and the steady state.

- (a) The depreciation rate falls.
- (b) The production function is Cobb-Douglas and capital's share rises.
- (c) Workers exert more effort so that they do in 50 minutes what it took 60 minutes to do.

Do the above for continuous time *and* discrete time.

## 3 Ramsey-Cass-Koopmans Model.

Suppose the economy is at balanced growth, but that at time  $t_0$  a tax rate  $\tau$  on capital income is unexpectedly instituted. Assume that the proceeds of the tax are distributed to people as lump-sum transfers.

- (a) What is the after-tax rate of return to the household?
- (b) How does the tax rate affect the  $\dot{q} = 0$  locus? The  $\dot{k} = 0$  locus?
- (c) How does the new balanced growth path compare to the old?

## 4 Question 3 Continued

In problem 3, assume that there are several economies like this. Preferences and technologies are the same, but tax rates differ. Assume that each country is in balanced growth. Assume that tax revenue is rebated on a lump-sum basis.

- (a) Show that the saving rate  $(y^* - c^*)/y^*$  is decreasing in  $t$ .
- (b) Do people in low  $\tau$ , high  $k^*$  countries have any incentive to invest in low saving countries?
- (c) How are your answers affected if there is a subsidy rather than a tax, i.e.  $\tau < 0$ ?
- (d) How are your answers affected if the government uses the tax revenue for public expenditure rather than rebates?

## 5 Leontief Production Function

$$C + Z = Y = \min[aK, bL]$$

- (a) Write down the intensive production function and plot  $y$  on the vertical axis versus  $k$  on the horizontal axis.
- (b) Using this production function, study the full dynamics of (i) the Solow model in continuous time, (ii) the Solow model in discrete time, (iii) the Ramsey-Cass-Koopmans model in continuous time.

## 6 Optimal Growth: Continuous Time

Consider the usual RCK optimal growth model except that the maximand is not smooth, viz.

$$\int_0^{\infty} u(c)e^{-\delta t} dt,$$

where  $u(c) = -\infty$  for  $c < \bar{c}$  and  $u(c) = c$  for  $c \geq \bar{c}$ ;  $\bar{c} > 0$  can be interpreted as the minimum consumption to sustain life.

- (a) Plot  $u(c)$ .
- (b) Describe the full optimal growth dynamics.
- (c) Describe the turnpike property.

The marginal product of capital is 8% per year. Capital's share (of output) is 25%. The rate of depreciation of capital is 15% per year. The population is growing at 1% per year. What do you know about this economy?