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Economics 614: Macroeconomics II
Spring 2005
Cornell University
Problem Set #9
Due: Friday, April 1, 2005

1 Inventive Activity and Capital Accumulation

$$\begin{aligned} Y &= Y_C + Y_I + Y_R = AK \\ Y_R &= (1/5)Y \\ Y_C &= (9/10)(4/5)Y \\ Y_I &= (1/10)(4/5)Y \\ \dot{K} &= Y_I - (1/5)K \\ \dot{A} &= Y_R - (1/20)A \end{aligned}$$

Fully describe the dynamical system including:
(a) the phase diagram
(b) the associated linear system
(c) the economic implications
(d) stability of the system
(e) robustness of the assumptions.

Change the production function to $Y = AK^{1/3}L^{2/3}$. Let $L = 1$. Redo the analyses above.

Change the production function to $Y = A^{1/5}K^{1/3}L^{2/3}$. Let $L = 1$. Redo the analyses above. (In parts 1 and 2, the production function exhibits increasing returns in the reproducible factors [1+1>1, $1 + 1/3 > 1$], while in part 2, the production function exhibits decreasing returns to the reproducible factors [$1/5 + 1/3 < 1$].)

2 Externalities.

Assume that the production function for firm i is

$$Y_i = F(K_i, K, L_i) = 72(K_i)^{1/3}(K)^{1/10}(L_i)^{2/3}$$

where K_i is capital employed in firm i, and K is the aggregate capital stock. Firm i is so small that its affect on K is negligible, but firm i believes that all firms act like themselves. Assume that F is homogeneous of degree one in K_i and L_i. Set up the RCK model for this situation. Solve it. Analyze it.