Lecture Notes #5

IDEAS

Ideas→Non-rivalry→IRS

→Non-competitive →Public Financed
→Imperfect Competition (including Patents)

Excludability -depends in part on transactions-cost

Romer Model

- Romer, P "Endogenous Technological Change" Journal of Political Economy, October 1990, 98(5): S71-S102
- Dixit and Stiglitz "Monopolistic Competition and Optimum Product Diversity" The American Economic Review, Vol. 67, No. 3 (Jun., 1977), pp. 297-308

$Y = K^{\alpha}(AL_Y)^{1-\alpha} \qquad \dot{K} = s_K Y - \mu K \qquad \dot{L} = nL \qquad L_A + L_Y = L$

$\dot{A} = \theta A^{\phi} L_A^{\lambda} \qquad \phi > 0$:standing on shoulders

$\lambda < 1$:stepping on toes

Steady State

$g_y = g_k = g_A$

$g_A = \frac{\dot{A}}{A} = \frac{\theta L_A^{\lambda}}{A^{1-\phi}} = const$

Log-differentiating RHS yields: $\lambda \frac{\dot{L_A}}{L_A} - (1-\phi)\frac{\dot{A}}{A} = 0$

$\lambda n - (1-\phi) g_A = 0$

$g_A = \frac{\lambda n}{(1-\phi)}$

Special Case #1

$\lambda = 1$ and $\phi = 0$

Productivity of researchers $=$ const θ

Hence :$\dot{A} = \theta L_A \qquad g_A = n$

Special Case #2

$\lambda = 1$ and $\phi = 1$

$$\dot{A} = \theta L_A A \qquad \frac{\dot{A}}{A} = \theta L_A$$

too rapid? See Shell & Romer

Final Goods Sector

$Y = L_Y^{1-\alpha} \int_0^A x_j^\alpha dj \quad p_Y = 1$

Competitive Profit Maximizing Producers (of Y)

$\max\limits_{L_Y, x_j} L_Y^{1-\alpha} \int_0^A x_j^\alpha dj - wL_Y - \int_0^A p_j x_j dj$

P_j is the rental price for capital good j and w is the wage rate

FOC: $w = \frac{(1-\alpha)Y}{L_Y} \qquad p_j = \alpha L_Y^{1-\alpha} x_j^{\alpha-1}$

Intermediate Goods Sector

Firm j purchases at a fixed cost the patent to produce capital of type j

from the new output Y on one-to-one basis

Firm j problem: $\max_{x_j} \pi_j = p_j(x_j)x_j - rx_j$

FOC (by symmetry): $p'(x)x + p(x) - r = 0$

$$p = \frac{1}{1+p'(x)x/p} r \text{ or } p = \frac{r}{\alpha}$$

$\frac{1}{\alpha}$ is the mark-up over MC, r

Intermediate Goods Sector

$$x_j = x \quad \pi_j = \pi \quad \pi = \alpha(1-\alpha)\frac{Y}{A}$$

Supply of capital=demand for capital:

$$\int_0^A x_j^{\alpha} dj = K \text{ or } x = \frac{K}{A}$$

Using $x_j = x$, we have:

$$Y = AL_Y^{1-\alpha} x^{\alpha} = AL_Y^{1-\alpha} A^{-\alpha} K^{\alpha} = K^{\alpha}(AL_Y^{1-\alpha})$$

Research Sector

Asset-Market Clearing Equation

Owner of 1 unit of K earns r: There is "no capital gain

on K" because $p_Y = 1$

Capitalist could also buy a patent at price p_A earning profit π / p_A and

expecting the capital gain $\dot{p}_A / p_A$

Asset markets do not clear unless: $(\pi + \dot{p}_A) / p_A = r$

Qusi-rents plus capital gains per dollar are equalized.

Assuming perfect foresight

In balanced growth: $p_A = \frac{\pi}{r-n}$

This is the price of a patent on the balanced growth path.

Labor market: $w_Y = \frac{(1-\alpha)Y}{L_Y}$ $w_R = MP_{L_A} * p_A$

In equilibrium: $w_Y = w_R = w$

Hence (after calculations) we have:

$s_R = \frac{1}{1+\frac{r-n}{\alpha g_A}}$ $r = \alpha^2 Y/K$ $MP_K = \alpha Y/K$ $r < MP_K$