Lecture Notes #6

Taxes and Transfers Denominated in Money: Static Case

- Balasko and Shell,"Lump-Sum Taxation: The Static Economy",in General Equilibrium, Growth, and Trade: The Legacy of Lionel McKenzie, II. (R. Becker, M. Boldrin, R. Jones and W. Thomson, eds.) New York: Academic Press, 1993, 168-180.

$$\max_{x_h} u_h(x_h)$$

$$s, t, \quad p \cdot x_h = p \cdot \omega_h - p^m \tau_h$$

$$p = (p^1, ..., p^i, ..., p^l) \in R^l_{++}, \ p^1 = 1$$

$$x_h \in R^l_{++}, \ \ \omega_h \in R^l_{++}, \ \ \tau_h \in R, \ \ p^m \in R_+$$

n individuals: $h = 1, 2, ...n$

a fiscal policy $\tau = (\tau_1, ..., \tau_h, ...\tau_n) \in R^n$ is said to be balanced

if $\sum_{h=1}^{n} \tau_h = 0$

a fiscal policy $\tau = (\tau_1, ..., \tau_h, ...\tau_n) \in R^n$ is said to be bonafide

if there is an equilibrium in which $p^m > 0$

Equilibrium:

$(p, p^m) \in R^l_{++} \times R_+$ in which $\sum_{h=1}^n x_h = \sum_{h=1}^n \omega_h$

Proper Monetary Equilibrium:

$(p, p^m) \in R^l_{++} \times R_{++}$ in which $\sum_{h=1}^n x_h = \sum_{h=1}^n \omega_h$

Proposition: every bonafide τ is balanced

Proof:

$p \cdot x_h = p \cdot \omega_h - p^m \tau_h$ Summing over h yields:

$p \cdot \sum_{h=1}^{n} x_h = p \cdot \sum_{h=1}^{n} \omega_h - p^m \sum_{h=1}^{n} \tau_h$

In equilibrium: $\sum_{h=1}^{n} x_h = \sum_{h=1}^{n} \omega_h$

Hence : $p^m \sum_{h=1}^{n} \tau_h = 0$. That is $p^m = 0$ or $\sum_{h=1}^{n} \tau_h = 0$

Proposition: for nice utility functions, every balanced fiscal policy is bonafide

Proof:

Tax-adjusted endowment $\widetilde{\omega}_h = (\widetilde{\omega}_h^1, ..., \widetilde{\omega}_h^l)$.

$\widetilde{\omega}_h^1 = \omega_h^1 - p^m \tau_h$, $\widetilde{\omega}_h^i = \omega_h^i \quad i = 2, ...l$

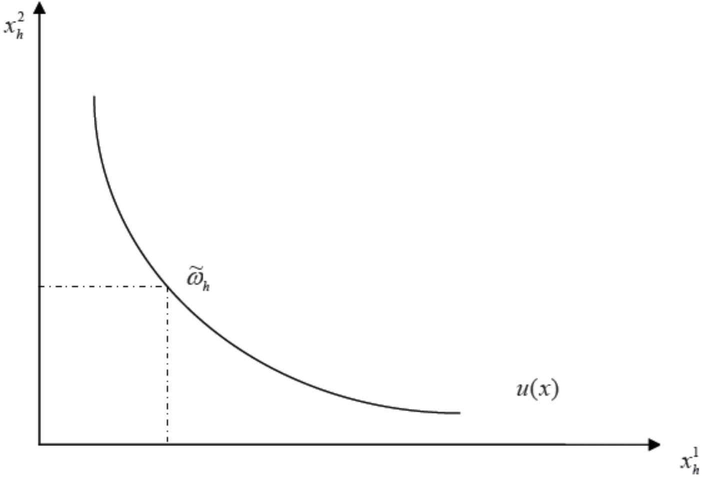
$w_h = p \cdot \omega_h - p^m \tau_h$

$p \cdot \widetilde{\omega}_h = p^1 \omega_h^1 - p^1 p^m \tau_h + p^2 \omega_h^2 + ... + p^l \omega_h^l \quad p^1 = 1$

$p \cdot \widetilde{\omega}_h = w_h = p \cdot \omega_h - p^m \tau_h$

$$\tau_h < 0 \Rightarrow \widetilde{\omega}_h \in R^l_{++}$$

$$\tau_h > 0 \Rightarrow \text{for small } p^m,\ \widetilde{\omega}_h \in R^l_{++}$$



Extension to 2 periods

$$\omega_h = (\omega_h^{1,1}, ..., \omega_h^{1,l}, \omega_h^{2,1}, ..., \omega_h^{2,l}) \in R_{++}^{2l}, \quad x_h \in R_{++}^{2l}, \quad p \in R_{++}^{2l},$$

$$\max_{x_h} \; u_h(x_h)$$

$$s, t, \quad p^1 \cdot x_h^1 + p^2 \cdot x_h^2 + p^{m,1} x_h^{m,1} + p^{m,2} x_h^{m,2} = p^1 \cdot \omega_h^1 + p^2 \cdot \omega_h^2$$

$$-p^{m,1}\tau_h^1 - p^{m,2}\tau_h^2$$

$$x_h^{m,1} + x_h^{m,2} = 0 \quad \text{so} \quad x_h^{m,1} = -x_h^{m,2}$$

Present Price of Money is Constant

$p^{m,1} = p^{m,2} = p^m \in R_+$

Proof:

We have $p \cdot x_h + (p^{m,1} - p^{m,2})x_h^{m,1} = p \cdot \omega_h - p^{m,1}\tau_h^1 - p^{m,2}\tau_h^2$

If $p^{m,1} > p^{m,2}$ then consumers will choose $x_h^{m,1}$ very negative to get unbounded utility. Sell High, Buy Low.

If $p^{m,1} < p^{m,2}$ then consumers will choose $x_h^{m,1}$ to be very large.

Buy Low, Sell High!

$x_h^{m,t} > 0$ h lends at t. $x_h^{m,t+1} < 0$ h borrows at t.

Ricardo and budget constraint

$$p \cdot x_h = p \cdot \omega_h - p^m \tau_h^1 - p^m \tau_h^2 = p \cdot \omega_h - p^m \mu_h \qquad \mu_h = (\tau_h^1 + \tau_h^2)$$

Timing of taxes foes not matter, only their present value

International finance

2 consumers, R for red, and B for blue. Static $I = 1$

$$px_h = p\omega_h - p^R\tau_h^R - p^B\tau_h^B$$

$$p\sum x_h = p\sum \omega_h - p^R\sum \tau_h^R - p^B\sum \tau_h^B$$

$$p^R\sum \tau_h^R + p^B\sum \tau_h^B = 0$$

Bonafide if $\sum \tau_h^R = \sum \tau_h^B = 0$

Bonafide if $p^R\sum \tau_h^R + p^B\sum \tau_h^B = 0$

Assume $\sum \tau_h^R \neq 0$, then for p^R and p^B to be positive

sign($\sum \tau_h^R$)$\neq$sign($^B\sum \tau_h^B$)

Exchange rate determined by

$$(p^R / p^B) = -(\sum \tau_h^B / \sum \tau_h^R)$$

Units. $R = \$$, $B = €$, $x =$ chocolate

$p^R =$ chocolates/\$ $\qquad p^B =$ chocolates/€

$\tau_h^R = \$ \qquad p^B = €$

$$\frac{chocalates/\$}{chocalates/€} = \frac{€}{\$}$$

$$\frac{€}{\$} = \frac{€}{\$}$$

Overlapping Generations

Example: 1 person per generation   2 periods   1 commodity per period

$\omega_0 = \omega_h^1 = 1 \qquad x_0 = x_0^1$

$\omega_t = (\omega_t^t, \omega_t^{t+1}) = (1, 1) \quad x_t = (x_t^t, x_t^{t+1}) \quad \text{for } t = 1, 2, ...$

$p = p^1, p^2, ..., p^t, p^{t+!}, ... \qquad p^{m,t} = p^m \geq 0$

$px_h = p\omega_h - p^R \tau_h^R - p^B \tau_h^B$

$p \sum x_h = p \sum \omega_h - p^R \sum \tau_h^R - p^B \sum \tau_h^B$

$p^R \sum \tau_h^R + p^B \sum \tau_h^B = 0$

Bonafide if $\sum \tau_h^R = \sum \tau_h^B = 0$

date / Individual	1	2	3			t	t+1
0	1						
1	1	1					
2		1	1				
t					1	1	
t+1						1	1

$$u_0(x_0) = u_0(x_0^1) = x_0^1 \quad u_t(x_t) = u_t(x_t^t, x_t^{t+1}) = x_t^t + x_t^{t+1}$$

Non-monetary CE is autarky

$$x_t = \omega_t \quad t = 0, 1, \ldots \quad \text{with } p^t = p^1 = 1 \quad t = 1, 2, \ldots$$

Interest rate $= 0$ CE is not PO. Over-saving!

date / Individual	1	2	3			t	t+1
0	1						
1	1 ↑	1					
2		1 ↑	1				
			↑				
t					1	1	
t+1						1 ↑	1

Chain Letter ! Ponzi Game

Infinite Horizon allows for CE which are not PO,

even though they are WPO and SRPO. Hilbert's Hotel

Money and OG

Monetary reform!

$(\tau_t^t + \tau_t^{t+1})$ can make everyone better off than in non-monetary CE