

Economics 614: Macroeconomics

Spring, 2010

Cornell University

Problem Set #1

Due: Friday, February 12, 2010

1. One-Sector Production.

$$\begin{aligned} Y &= F(K, L), \\ C + Z &= Y \end{aligned}$$

$$K > 0, L > 0, C \geq 0, Z \geq 0$$

(a) Given K and L , what is the PPF (production possibility frontier)? Be precise.

Let $F(K, L) = 4K^{\frac{1}{4}}L^{\frac{3}{4}}$. Assume perfect competition.

(b) What do we know about the competitive shares (of the output) for capital and labor?

(c) How is the composition of output affected by output prices?

(d) If $(K, L) = (5, 5)$, what are the rental rate on capital (r) and the wage rate for labor (w)? These are the input (or factor) prices.

The ratio $\omega = \frac{w}{r}$ is the factor price ratio; let $k = \frac{K}{L}$.

(e) How does ω vary with k ?

2. One-Sector Production.

(a) Show that if the production function is

$$AK^\alpha L^{1-\alpha},$$

where $0 < \alpha < 1$, that capital's share is α and labor's share is $1 - \alpha$.

(b) Show that if capital's share is α and labor's share is $1 - \alpha$, then the production function is

$$AK^\alpha L^{1-\alpha},$$

where $A > 0$ is an undetermined constant.

3. One-Sector Production.

Cobb and Douglas found that factor shares are constant. Later studies suggest that labor's share is increasing over time (while capital intensity is increasing). What does this suggest about the possible specification of the aggregate production function?

4. Two-Sector Production.

I. For each of the following:

(a) draw the Harrod-Johnson diagram

and

(b) calculate the incomplete specialization output price ratios, \bar{p} and \underline{p} and indicate what happens to these as k (the overall capital intensity) is varied.

(1)

$$\begin{aligned} Y_1 &= 4K_1^{3/4}L_1^{1/4} \\ Y_2 &= 10K_2^{1/2}L_2^{1/2} \\ K &= 100, L = 20 \end{aligned}$$

(2)

$$\begin{aligned} Y_1 &= 5K_1^{.5}L_1^{.5} \\ Y_2 &= 10K_2^{.5}L_2^{.5} \\ K &= 30, L = 3 \end{aligned}$$

(3)

$$\begin{aligned} Y_1 &= K_1^{1/2}L_1^{1/2} \\ Y_2 &= K_2^{3/4}L_2^{1/4} \\ K &= 12, L = 12 \end{aligned}$$

(4)

$$\begin{aligned} Y_1 &= K_1^{3/4}L_1^{1/4} \\ Y_2 &= K_2^{3/4}L_2^{1/4} \\ K &= 15, L = 15 \end{aligned}$$

II. Describe the role of capital intensities in the 2-sector model. When is $\underline{p} < \bar{p}$?
When is $\underline{p} = \bar{p}$?

III. If two countries have the same output price ratio, do they have the same factor price ratio?