

Professor Karl Shell
March 29, 2004

Economics 614: Macroeconomics II

Spring 2004

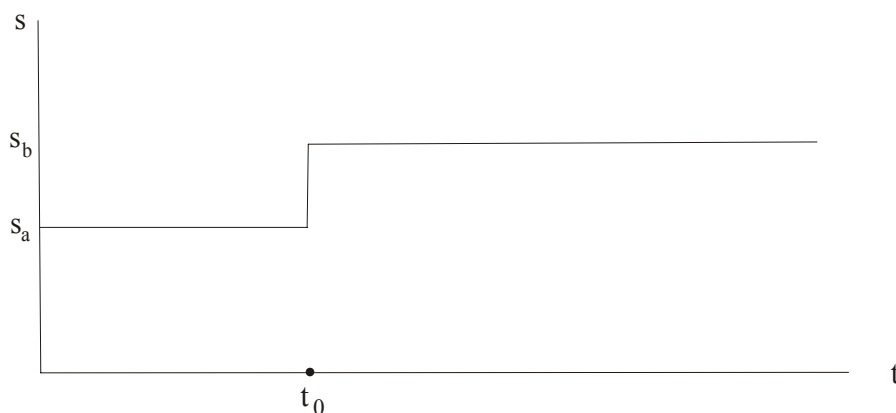
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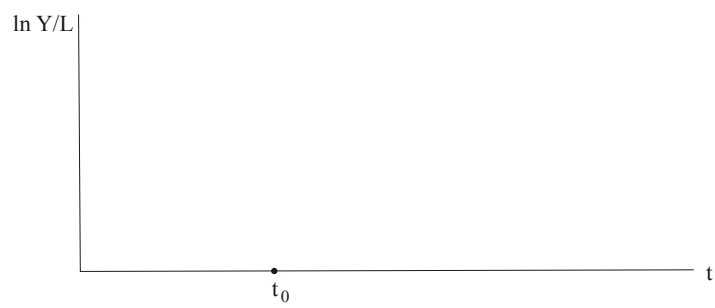
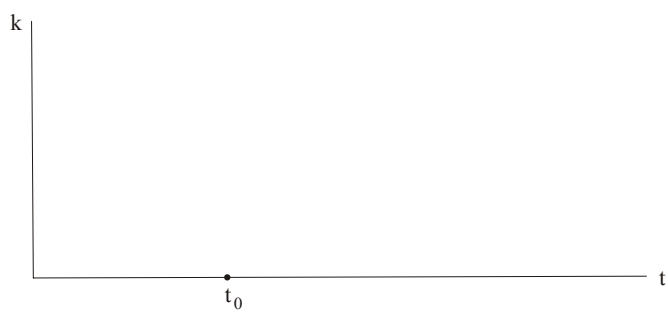
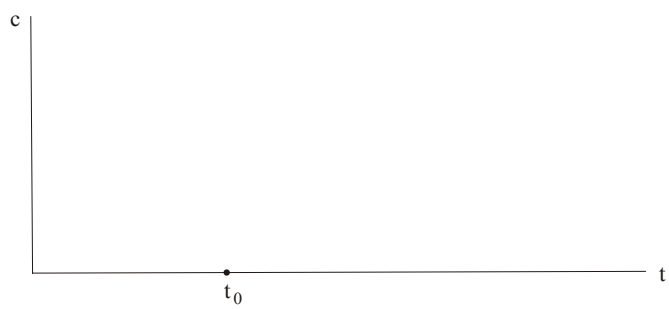
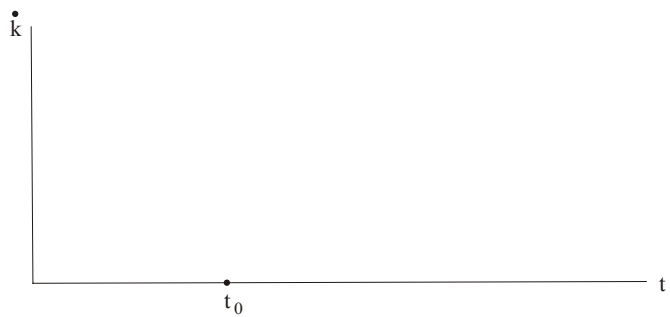
Problem Set #7

Due: Monday, April 5

1 Solow Growth Model, Continuous Time.

Impulse diagrams. Suppose the economy is in a steady-state, but at time t_0 the savings rate jumps from s_a to s_b , where $s_b > s_a$ (as below). Complete the graphs on the following page.





2

Describe how the following affects the Solow diagram and the steady state.

- (a) The depreciation rate falls.
- (b) The production function is Cobb-Douglas and capital's share rises.
- (c) Workers exert more effort so that they do in 50 minutes what it took 60 minutes to do.

Do the above for continuous time *and* discrete time.

3 Ramsey-Cass-Koopmans Model.

Suppose the economy is at balanced growth, but that at time t_0 a tax rate τ on capital income is unexpectedly instituted. Assume that the proceeds of the tax are distributed to people as lump-sum transfers.

- (a) What is the after-tax rate of return to the household?
- (b) How does the tax rate affect the $\dot{q} = 0$ locus? The $\dot{k} = 0$ locus?
- (c) How does the new balanced growth path compare to the old?

4

In problem 3, assume that there are several economies like this. Preferences and technologies are the same, but tax rates differ. Assume that each country is in balanced growth. Assume that tax revenue is rebated on a lump-sum basis.

- (a) Show that the saving rate $(y^* - c^*)/y^*$ is decreasing in t .
- (b) Do people in low τ , high k^* countries have any incentive to invest in low saving countries?
- (c) How are your answers affected if there is a subsidy rather than a tax, i.e. $\tau < 0$?
- (d) How are your answers affected if the government uses the tax revenue for public expenditure rather than rebates?

5 Externalities.

Assume that the production function for firm i is $Y_i = F(K_i, K, L_i)$ where K_i is capital employed in firm i , and K is the aggregate capital stock. Firm i is so small that its affect on K is negligible, but firm i believes that all firms act like themselves. Assume that F is homogeneous of degree one in K_i and L_i . Set up the RCK model for this situation. Solve it. Analyze it.

6 Oversaving.

Labor's share is 33%. With one unit of capital and one unit of labor, output is one unit. The depreciation rate is 10%. The population growth rate is 2%. Describe precisely the inefficient growth paths, the efficient growth paths, and paths that are not easily recognized as efficient or inefficient.

7 Diamond Model.

Analyze the dynamics and steady state when:

- (a) the utility function is logarithmic and the production function is Cobb-Douglas.
- (b) the utility function is logarithmic and the production function is CES.

8 Social Security.

Consider the Diamond model where production is Cobb-Douglas and utility is logarithmic.

- (a) Pay-as-you-go. Suppose the government taxes each young person and proceeds to pay benefits to old people. Suppose each old person receives $(1+n)T$. (Taxes can be taken as lump-sum.)
 - (i). How does this change the law of motion for k^t to k^{t+1} ?
 - (ii) How does this affect the balanced growth path?
 - (iii) If the economy is dynamically efficient, how does a small increase in T affect the welfare of current and future generations?
- (b) Fully-funded system. Suppose the government taxes each young person T and uses the proceeds to purchase capital. A person from generation t receives $R^{t+1}T$ when old.
 - (i) How is the law of motion from k^t to k^{t+1} affected?
 - (ii) How does this affect the steady state?