Sunspot Equilibrium

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The volatility of market outcomes such as the price level, stock market prices, unemployment rates, interest rates, and exchange rates and what to do about this are important subjects in macroeconomics. Some of the observed randomness in market outcomes is the result of shocks to the fundamentals (preferences, technologies, and endowments) that are transmitted through the economy. Uncertainty about the economic fundamentals is *intrinsic uncertainty*. The general-equilibrium model extended by Arrow (1953, 1964) to include uncertainty provides an explanation of how volatility in the fundamentals is transmitted through the economy, resulting in volatile prices and quantities. This is not the only possible source of the volatility in economic outcomes. The market economy is a social system. In attempting to optimize her own actions, each agent must attempt to predict the actions of the other agents. A, in forecasting the market strategy of B, must forecast B's forecasts of the forecasts of others including those of A herself. An entrepreneur is uncertain about the moves of his customers and his rivals, and they of his moves. It is not surprising that this process may generate uncertainty in outcomes even in the extreme case in which the fundamentals are non-stochastic. The uncertainty generated by the economy is *market uncertainty*. It is either created by the economy or adopted from outside the economy as a means of coordinating the plans of individual agents. Market uncertainty is not transmitted through the fundamentals. It can be driven by *extrinsic uncertainty*.

'Sunspots' is short-hand for 'the extrinsic random variable' (or 'extrinsic randomizing device') upon which agents coordinate their decisions. In a proper sunspot equilibrium, the allocation of resources depends in a non-trivial way on sunspots. In this case, we say that *sunspots matter*; otherwise, sunspots do not matter. Sunspot equilibrium was introduced by Cass and Shell; see Shell (1977) and Cass and Shell (1982, 1983). Sunspot models are complete rational-expectations, general-equilibrium models that offer an explanation of excess volatility. It was by no means a new idea that economies can and do generate excess volatility, but the sunspots model is the first general-equilibrium model to exhibit excess volatility even when agents are fully rational. The sunspots model also allows for non-rational agents, but the excess volatility from this source – while possibly empirically substantial – is less novel.

'Sunspots' is a spoof on Jevons (1884), who in serious empirical work attempted to explain the business cycle by relating it to the observed (through telescopes) cycle of actual sunspot activity. To the extent that actual sunspot activity does affect economic fundamentals (such as crop yields and cancer risk), this is an instance of intrinsic uncertainty, but the effects of actual sunspots on fundamentals are probably very small. If actual sunspots have only a minor effect on the fundamentals, but they do have a substantial effect on the economy, it must be that actual sunspots serve a role in the economy beyond their effects on the fundamentals. Manuelli and Peck (1992) show that a sunspot equilibrium can be interpreted as the limit of traditional rational-expectations equilibria as the uncertainty in the fundamentals vanishes. See also Spear, Srivastava, and Woodford (1990). Roughly speaking, 'Jevons equilibrium' becomes'Cass-Shell equilibrium' as the effects of actual solar activity on the fundamentals go away. Cass-Shell sunspot equilibria are easy to interpret because in the basic sunspots models the only uncertainty is extrinsic uncertainty. Hence any volatility in outcomes is excess volatility. Engineers compute 'gain' in noise as the volatility of the output signal divided by the volatility of the input signal. In a sunspot equilibrium, the gain is $+\infty$.

The first sunspots model, Shell (1977), is of an overlapping-generations exchange economy with taxes and transfers denominated in fiat money. This OG model is based on the very simple (degenerate) example used in Shell (1971) to show that restrictions on market participation are inessential in the Samuelson perfect-foresight (non-stochastic) OG model. The only stochastic element in the 1977 paper is sunspot-driven extrinsic uncertainty about the price level. Shell used the fact that there is a continuum of equilibria (parameterized by the initial price level) in the non-sunspots version to construct by a bootstrap method the sunspot equilibrium allocation. This particular sunspot equilibrium, while bootstrapped from multiple certainty equilibria, is *not* a mere randomization over certainty equilibria contrary to what some popularizers of sunspot equilibrium have claimed. Sunspot equilibria can be mere randomizations over certainty equilibria, but typically they are not. In unpublished work in about 1975, Cass and Shell generalized the OG sunspots analysis from the degenerate linear model to the concave-utility-function OG model of Gale (1973). Peck (1988) showed that for the 'Samuelson' and related cases in the OG economy, sunspots can be active in every period for economies with even non-stationary environments.

Azariadis (1981) translated the pure-exchange OG model into a macro-oriented Lucas-style OG model with capital investment and endogenous labor supply. Azariadis showed instances – based on a backward-bending offer curve – of economies which exhibit long-run stationary sunspot cycles. Azariadis and Guesnerie (1986) employed the backward-bending offer curve to exhibit economies with long-run deterministic cycles. They thus established a link between sunspot cycles and deterministic cycles. Roughly, if there is room to condition expectations on sunspots there is also room to condition them on calendar time and *vice versa*. See Cass and Shell (1980).

To get a better feeling for how sunspot equilibria arise, consider two simple, related examples. In the first, the economy is immune from sunspots. In the second, all competitive equilibria are sunspot equilibria. Consider the two-consumer, one-good, two-states-of-nature, competitive exchange economy. Draw the Edgeworth box. Measure consumption in state α ('sunspots') on the horizontal and consumption in state β ('no sunspots') on the vertical. Endowments lie on the minor diagonal, because endowments are by definition independent of the state of nature. For the same reason, the Edgeworth box is a square. Assume that consumers possess smooth, strictly concave von Neumann-Morgenstern utility functions. Competitive equilibrium exists. There are two cases: (1) Consumers share the same probability beliefs about the occurrence of sunspots. Indifference curve tangency and hence competitive equilibrium occurs only on the minor diagonal with contingent claims prices proportional to the probabilities. Sunspots do not matter. This is an instance of the *Cass-Shell Sunspot Immunity Theorem* (1983, Proposition 3). It holds when the box is square, i.e. whenever there is no aggregate uncertainty. (2) Consumers differ in their beliefs. Indifference curves will be tangent to each other but always off the minor diagonal. Sunspots matter.

Heterogeneity of (probability) beliefs is a source of sunspot equilibria, but it is hardly the only source. The Sunspots Immunity Theorem is based on a finite model with strictly convex preferences and convex production and a full range of perfectly competitive markets. Any departure from these assumptions could be a possible source of sunspots that matter. (See Shell 1987, page 550, for the so-called Philadelphia Pholk 'Theorem' on how to find sunspots that matter.) For example, the usual overlapping-generations models (including the one in Shell, 1977) fail to fit the assumptions of the Immunity Theorem in three ways:

1. *There are natural restrictions on participation in the securities markets.* If a random variable is realized before your birth, you cannot buy securities dependent on its realization. See Cass and Shell (1983) for analysis of sunspot equilibria caused solely by restricted market participation. Balasko, Cass and Shell (1995) also focus on restrictions on market participation. If there are no (or sufficiently few) restricted agents, then sunspots do not matter in convex, finite economies. If all individuals are restricted, then sunspot equilibria are mere randomizations over nonsunspot equilibria. Otherwise, the typical sunspot equilibrium is not a mere lottery over nonsunspot equilibria.

2. *The securities market is incomplete.* There is only one money. Completeness of the market would require instead state-contingent Arrow securities for each state of nature at each date. General equilibrium with incomplete markets, sometimes studied under the acronym GEI, is an important area in financial economics that was spawned by the sunspot-equilibrium model. Cass (1989, 1992), Balasko and Cass (1989), and others have played central roles in developing the GEI model and placing it in the sunspots-equilibrium literature. It is worth noting, however,

that incomplete markets do not necessarily lead to sunspot equilibria; see, for example, Antinolfi and Keister (1998), who show that with only a few options (puts and calls) with the right strike prices the economy can be immune from sunspots even when there are many sunspot states of nature.

3. *The OG model is not a finite model.* There are a countable number of individuals and a countable number of dated commodities. There can be sunspot equilibria in the OG economy even if we assume that markets are completed with Arrow securities for every state and every date, and that, contrary to actual biology and demography, agents are not restricted in the trades of these securities. In this thought-experiment, they can even buy securities to hedge against events that occur before their natural lifetimes. See Cass and Shell (1989). The unbounded horizon permits bubbles in the form of public debt that need not be retired. If there can be a bubble in an infinite-horizon economy, then it is likely that there can also be a stochastic (or sunspot) bubble. The infinite horizon is in itself a source of sunspot equilibria. Sunspots can be an imperfect substitute for fiat money in the 'Samuelson' case of the OG model. See Cass and Shell (1989).

Each of these three departures from the finite, perfect-market competitive equilibrium economy is *in itself* a separate source of sunspot equilibria allocations. So the OG model is a natural and – it turns out – a relatively easy place to find sunspots that matter. It is also natural to expect that non-convexities create a role for sunspot equilibria. Random allocations would seem to offer the possibility of at least partially 'convexifying' the certainty economy. This turns out to be the case. Shell and Wright (1993) analyze sunspot equilibrium in competitive, exchange economies with an indivisible good. They show that the Rogerson (1988) indivisible-labor lottery equilibrium can be decentralized as a sunspot equilibrium even in finite economies as well as in continuum-of-agents economies. Unlike the situation in the finite, convex economy: (1) Sunspot equilibrium allocations in non-convex economies are Pareto optimal among stochastic allocations and often strictly dominate the best allocations available in the economy that does not have access to randomization. (2) The certainty allocations do not necessarily re-appear in the sunspots model as non-sunspot equilibria. Goenka and Shell (1997b) extend the Shell-Wright analysis to non-convex production. See also Goenka and Shell (1997a). An earlier paper on sunspots in non-convex economies is Guesnerie and Laffont (1991). Indivisible labor and sunspots are central to a recent contribution to the theory of money and search; see Rocheteau, Rupert, Shell, and Wright (forthcoming). Previous monetary search models have required for tractability the apparently restrictive assumption that agents possess quasi-linear utility functions. If one assumes that labor is indivisible and is allocated to work or leisure by a sunspot process, then von Neumann-Morgenstern agents act as if their utilities are quasi-linear.

What is the relationship between the sunspot-equilibrium concept and the lottery-equilibrium concept introduced by Prescott and Townsend (1984a, 1984b)? The original motivations for the two concepts were very different. The first sunspots papers focused on stochastic allocations that are Pareto non-optimal, cases where sunspots lead to inefficient allocations because of restrictions on market participation, incomplete markets, the infinite horizon, or imperfect competition. The first Prescott-Townsend lottery equilibrium papers focused on random allocations that partially remedy the effects of 'non-convexities' in the certainty economy due to moral hazard constraints. Because sunspots form the basis for coordination of individual plans, the sunspot equilibrium notion is directly applicable in economies with few agents, many agents, or even a continuum of agents. The original lottery equilibrium notion was applicable only to economies with a continuum of agents, in which detailed coordination is not necessary because of the law of large numbers. An important formal difference between these two stochastic equilibrium concepts is based on how commodities are defined. In the sunspots model, the commodity might be chocolate delivered in state α. In the lottery model, the commodity might be chocolate delivered with probability π. If the sunspot random variable used in each case is continuous (i.e., has a non-atomic density function), then lottery equilibrium allocations are always sunspot equilibrium allocations. See Garratt, Keister, Qin, and Shell (2002). For lottery equilibria to make sense in finite economies (without the law of large numbers), the lottery equilibrium notion must be suitably adjusted as in Garratt (1995) to ensure that in equilibrium materials balance for every realization of the randomizing device. After making the Garratt adjustment, it is shown by Garratt, Keister, Qin, and Shell (2002) that for economies with a finite number of agents *or* a continuum of agents, and a continuous randomizing device, the set of sunspot equilibrium allocations is identical to the set of lottery equilibrium allocations. Garratt, Keister and Shell (2004) show that this equivalence does not always hold when the randomizing device is finite. Kehoe, Levine, and Prescott (2002) establish the equivalence of sunspot equilibrium allocations and lottery equilibrium allocations in economies with a continuum of agents facing 'non-convexities' caused by moral hazard constraints. Prescott and Shell (2002) provide a review of the sunspot and lottery literatures and attempt to highlight the relatively strong similarities and the non-trivial differences between the two concepts.

While the notion of sunspot equilibrium was not immediately accepted by macroeconomists, game theorists were not at all shocked by the idea of stochastic outcomes in non-stochastic environments. Think mixed strategy and – more generally – correlated equilibrium. Peck and Shell (1991) analyze in market games the relationship of sunspot equilibria to correlated equilibria defined by Aumann (1974, 1987). Peck and Shell show that every correlated equilibrium allocation can be decentralized as a sunspot equilibrium allocation, but the converse is not true. Correlated equilibria are self-enforcing while sunspot equilibria allow for transfer of incomes across states of nature. The market game is the leading general-equilibrium model of imperfect competition. It is shown by Peck and Shell that unless endowments are Pareto optimal, there is always a proper sunspot equilibrium due to imperfect competition. Imperfect and monopolistic competition are highly prone to sunspot effects. Imperfect competition is one of the very useful building blocks for calibrating applied sunspot models to business cycle data. Peck and Shell (1991) also incorporate asymmetric information into the sunspots model. Earlier examples of sunspot equilibria in which the randomizing device provides asymmetric (but correlated) information to agents are Azariadis and Guesnerie (1982) and Maskin and Tirole (1987).

Detailed market structure matters in imperfectly competitive economies. Peck and Shell (1989) construct two different securities games from the same certainty market game. In one, there is a full spectrum of Arrow financial securities. In the other, there is a full spectrum of real contingent commodities. A sunspot Nash equilibrium allocation in the Arrow securities game which is not a mere lottery over certainty Nash equilibrium allocations is *never* a Nash equilibrium allocation to the contingent-commodities game. The two games differ because the market power of individual agents depends on the way markets are organized.

Yves Balasko (1983) provides a general definition of extrinsic uncertainty. Modeling how extrinsic uncertainty affects technologies and endowments is straightforward: Endowments and input-output pairs are independent of the realization of the randomizing device. Modeling *ex-ante* preferences is more subtle. It is only required that if states of nature are renamed, say: α becomes β, the allocation in α becomes the allocation in β, and the probability $\pi(\alpha)$ becomes $\pi(\beta)$, then *ex-ante* utility is unaffected. This generalizes von Neumann-Morgenstern utility. Balasko (1990) shows how sunspot equilibrium is an instance of symmetry-breaking in economics. The non-sunspot equilibrium is the symmetric solution to symmetric equations. The sunspot equilibrium breaks the symmetry of endowments, technologies, and preferences. It is an asymmetric solution to the symmetric equations.

Benhabib and Farmer (1994), Farmer and Guo (1994), and Galí (1994) launched the field of applied sunspot business cycle analysis. See also Benhabib and Farmer (1996), Benhabib and Nishimura (1998), and Benhabib and Wen (2004). They made only simple adjustments to the standard real business cycle model of Kydland and Prescott (1982) in their set-ups. For example, Benhabib and Farmer (1994) – following the lead of Spear (1991) – introduced an externality in production leading to aggregate increasing returns to scale. Without this adjustment, sunspots would not matter because the standard RBC model (based on a single infinite-lived individual) is equivalent to a planning model, so when preferences and technology are convex, sunspots cannot matter. With this adjustment, there can be multiplicity of certainty equilibrium paths which leads to the existence of equilibrium fluctuations driven by sunspots. Farmer and Guo (1994) calibrated a discrete-time version of the Benhabib-Farmer model to match business cycle facts while employing only sunspot uncertainty. Yi Wen (1998) replaced the Benhabib-Farmer externality with capacity utilization, reducing the size of the externality to a more reasonable level, while still matching business cycle facts without positing any intrinsic uncertainty. Calibration of sunspot-driven business cycles is a major and growing area. There is barely room in this review to scratch the surface. The applied sunspots business cycle calibrators were wise in deviating only in relatively small steps from the well-established RBC model for their experiments. Otherwise they would have been less likely to get the attention of the calibration community. On the other hand, one guesses that sunspot allocations will be easier to find and easier to match to data in the overlapping-generations economy.

Some sharp economic downturns have been attributed to ‘panics’ or ‘bursting bubbles’ in financial markets. People ‘run’ on a bank or other financial institution when they expect others to run. In their classic bank-run model, Diamond and Dybvig (1983) highlight the fragile nature of financial intermediaries. Banks attempt to smooth consumption between depositors who turn out to be patient (and can afford to wait) and those who turn out to be impatient (and need to withdraw early). The problem is that the 'patient' people might panic, attempting to withdraw early and causing a run on the bank. In the standard bank contract, there are two equilibria to the post-deposit game: (1) the (good) no-run equilibrium and (2) the (bad) run equilibrium. However, the run equilibrium is not an equilibrium for the pre-deposit game: If individuals know in advance that there will be a run on the bank, they will not make a deposit. Hence in the formal model, bank runs are not possible. Diamond and Dybvig suggest that sunspots will play a role in panic-based runs. Peck and Shell (2003, 2005) validate their intuition. It is shown that panic based bank runs can be part of a sunspot equilibrium for the pre-deposit game even when partial suspension of convertibility is allowed. Sunspot-driven bank runs are possible, but they are typically not mere randomizations over certainty equilibria. If the probability of the run is small, the optimal banking contract tolerates runs. If the probability of the run is large, then the optimal banking contract is run-proof. Ennis and Keister (2003) exploit these ideas to investigate the implications of the possibility of bank runs on economic growth. Gu (2006) considers an asymmetric-information, extrinsic randomizing device in the banking setup. If the sunspots signals are highly correlated, there exists a proper correlated equilibrium for some banking contracts. In the equilibrium, depending upon the realization of the signals, either a full bank run, or a partial bank run, or no bank run will occur.

What policies should be taken to stabilize or even immunize the economy from sunspot fluctuations? Complete immunization might not be feasible and when feasible it might not be desirable. For example, we know that while avoiding bank runs can be feasible, the optimal banking contract tolerates runs at small probabilities. See Ennis and Keister (2005) for some other examples. Grandmont (1985, 1986) designs government policies that immunize the economy from sunspot effects. Grandmont's policies set taxes according to feedback rules that render the current price level as predetermined and thus immunizing it from sunspots. Smith (1991b) considers the policy of inflation rate targeting in an overlapping-generation economy. If the government maintains a target price-level path by standing ready to exchange money for interest-bearing assets, this immunizes the economy from sunspots but at the cost of substantial inefficiency. Other policies that target a given price level lead to smaller inefficiencies even though they do not immunize the economy from sunspot effects. Woodford (1988) considers a cash-in-advance economy in which the government either fixes the constant money growth rate or fixes a constant nominal interest rate. Woodford finds that there is a unique equilibrium under the interest rate rule, but the corresponding constant money growth economy is susceptible to sunspot shocks. Woodford (1988) differs from Smith (1991b), because Woodford uses a transversality condition not appropriate in Smith's OG environment. For Woodford, under the interest rate rule all price histories but one result in too rapid accumulation of government debt. Keister (1998) studies a model with segmented asset markets in which the amount of sunspot-driven volatility in consumption depends on the government's tax-transfer policy. A policymaker concerned about inequality may choose to accept sunspot volatility in order to achieve some redistribution.

It has been proposed that narrow banks, banks that are restricted to holding only liquid assets, are more stable than wide banks, banks that are unrestricted in asset holding. Peck and Shell (2005) show that narrow banks are subject to sunspot-based panic runs while wide banks are immune from these. On the other hand, wide banks are subject to running out of funds in the face of intrinsic shocks, while the narrow banks are immune from these shocks because of their over-investment in the liquid asset. Goenka (1991, 1994a) shows that restrictions on government institutions intended to increase bureaucratic accountability can also increase the fragility of the economy in the face of sunspot shocks. For example, forcing government agencies to finance their separate budgets through agency-specific taxes can introduce sunspot instability and inefficiency.

Can sunspot equilibria be dismissed as less 'stable' or more 'fragile' than non-sunspot equilibria? The current answer to the question seems to be 'no', meaning sunspot equilibria cannot in general be dismissed. For example: (1) Woodford (1990) shows that under some plausible assumptions, the economy will learn to believe in sunspots. (2) Balasko, Cass and Shell (1989) show that if the parameters of the sunspot economy are slightly perturbed then sunspot equilibrium allocations will typically move to nearby sunspot equilibrium allocations. Of course, these stability results are only for specific models.

It has been possible to sketch only a few of the many excellent contributions to the very rich and extensive literature on sunspot equilibrium. Sunspots now play important roles in both descriptive and normative economics. They naturally arise in dynamic economies in which expectations play a central role. They matter when markets are incomplete or participation in them is restricted. They matter when the horizon is infinite. They matter when preferences and/or technologies are non-convex . … Sunspots matter.

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