stable. Because of nonlinearities, its stability depends on the initial conditions and the behavior of exogenous variables. Under normal circumstances, without well-designed discretionary policies, the system tends to exhibit fairly long (4 to 10 years) cycles whose amplitudes may gradually increase or decrease over time. Moreover, in the absence of such policies the system may drift away from the desired growth path or exhibit cycles undesirably large in amplitudes, but its response is slow enough so that well designed stabilization policies can control the system.

These results are for the MPS model and not for the actual economy. Even for the MPS model these findings are only preliminary. Nevertheless, to the extent that these characteristics are true of the economy, it suggests that the type of analysis Professor Tobin formulated and we have amplified in this paper, as well as other studies of the short-run dynamics of the economy, must be pursued with even more diligence in the future.

12.5. Appendix: demand for money in a general portfolio model in the presence of an asset that dominates money[15] *(by Albert Ando and Karl Shell)*

Suppose that there are three assets: equity, savings deposits, and money.[16] As in most of the literature on the transactions demand for money, assume that commodities must be paid for by money and some cost is incurred whenever money is exchanged with other financial assets.[17] We consider a two-period problem, and let consumption in each period be C_1 and C_2. The consumer starts out with an initial asset valued at W, of which he consumes C_1 during the first period. Consumption takes place evenly over period 1. He also maintains an average money balance M and average equity and savings deposits balances of E and S, respectively. Let the money price of consumption goods in period 1 be unity. Also let

$$
\begin{aligned}
E/(W-0.5C_1) &= e,\\
S/(W-0.5C_1) &= s,\\
M/(W-0.5C_1) &= m. \qquad (12.22)
\end{aligned}
$$

[15] This analysis is a preliminary result of a larger work by the authors.

[16] It is a trivial generalization to introduce more than one type of equities and many short-term securities into the model, as the structure of our argument will indicate.

[17] See, for instance, Tobin [5].

Under the assumption that all assets are consumed by the end of the second period, we have

$$C_2 = W - C_1 + (W - 0.5C_1)[e\tilde{\rho} + s(r_s - \tilde{p}) + m(r_m - \tilde{p})] - T(M, C_1), \tag{12.23}$$

where $\tilde{\rho}$, r_s, and r_m are the rate of return on equity, savings deposit, and money, respectively, and $\tilde{p}$ is the rate of change of price of consumption goods.[18] $\tilde{\rho}$ and $\tilde{p}$ are random variables, while r_s and r_m are known with certainty. In other words, returns on money and savings deposits are subject to the same uncertainty due to changes in the general price level. T is the real transactions cost incurred during period 1. The utility function becomes

$$U^*(C_1, C_2) = U^*\{C_1, W - C_1 + (W - 0.5C_1)[e\tilde{\rho} + s(r_s - \tilde{p}) + m(r_m - \tilde{p})] - T(M, C_1)\}. \tag{12.24}$$

If we now assume that C_1 is determined independently of portfolio choice, then we can define a simpler function,

$$U(C_1, C_2) = U\{W - C_1 + (W - 0.5C_1)[e\tilde{\rho} + s(r_s - \tilde{p}) + m(r_m - \tilde{p})] - T(M, C_1)\}, \tag{12.25}$$

and maximize its expected value over the distribution $\phi(\tilde{\rho}, \tilde{p})$ with respect to e, b, and m. Remembering that

$$e + s + m = 1,$$

we have

$$\max_{e,m} \underset{\tilde{\rho},\tilde{p}}{E}\{U[W - C_1 + (W - 0.5C_1) \times [e\tilde{\rho} + (1 - e - m)(r_s - \tilde{p}) + m(r_m - \tilde{p})] - T[(W - 0.5C_1)m, C_1]]\}, \tag{12.26}$$

[18] This formulation implies that transaction costs are paid at the end of the first period. If we assume that transaction costs are paid continuously as transactions take place, the analysis becomes a little more complicated, but the essential characteristics of the result can be maintained.

$$\partial E/\partial e = E\{(W-0.5C_1)[\tilde{\rho}-(r_s-\tilde{p})]U'\}=0, \tag{12.27}$$

$$\partial E/\partial m = E\{(W-0.5C_1)[-(r_s-\tilde{p})+(r_m-\tilde{p}-T_M(\,.\,))]U'\}=0.$$

Since $\tilde{p}$ cancels, we can rewrite this expression as

$$(W-0.5C_1)[(r_m-r_s)-T_M(.)]E(U')=0, \tag{12.28}$$

where $T_M(.)$ stands for the partial derivative of T with respect to $M=m(W-0.5C_1)$, and should be interpreted as the marginal reduction in transactions cost as money holding is increased by \$1. In the left-hand side of (12.28) it can be presumed that $W-0.5C_1 \neq 0$, and $E(U')\neq 0$. Hence, (12.28) can be simplified to

$$r_s-r_m=-T_M(M,C_1). \tag{12.29}$$

The condition (12.29) states that, under the conditions formulated, the demand for money is a function only of (r_s-r_m), C_1, and parameters of T, and independent of $\tilde{\rho}$ and $\tilde{p}$, and W. Turning it around, if $r_m=0$ as is usually assumed, and M is determined by monetary policy, r_s is determined in the money market alone, independent of supply of S, of the rate $\tilde{\rho}$, and of the rate of change of prices, $\tilde{p}$.

One important consequence of this analysis is that the division of W into E and $M+S$ can be considered almost independently of the demand for M, except in so far as the demand for M determines r_s, and r_s affects the division of W between E and $M+S$. For the result to hold strictly, m must enter (12.27) only in the form $(m+s)$, and r_m not at all. m and r_m enter the first of the above first-order conditions only through U'. We rewrite the argument of U' as

$$\begin{aligned} W-C_1+(W-0.5C_1)[e\tilde{\rho}-\tilde{p}(1-e)+r_s(1-e)]& \\ +(W-0.5C_1)m(r_m-r_s)-T(M,C_1).& \end{aligned} \tag{12.30}$$

In (12.30), e appears only in the second term while m and r_m appear only in the third and fourth terms. Hence, e is independent of m and r_m if the term

$$\delta=\Delta M(r_s-r_m)+\Delta r_m M-[T(M+\Delta M,C_1)-T(M,C_1)]$$

is zero. δ is the net change in the total cost of holding and managing the cash component of the portfolio, including the transactions cost, when M is changed due to changes in r_m or other reasons, given r_s and C_1. δ is unlikely to be zero, but it should be a very small fraction of C_2. Therefore, so long as U' is a reasonably smooth function of C_2 so that a small change in C_2 leaves U' nearly unchanged (that is, U' and U'' are both continuous in C_2), a change in demand for M given r_s will leave e nearly unaffected.

There is another reason why e should be nearly independent of m given r_s. Since δ is nonstochastic, δ can be thought of as having the same effect on e as a change in W, except for the secondary effects due to our assumption that transactions costs are paid at the end of the period and not continuously. Therefore, if U is such that e is independent of the size of W (u is 'constant risk aversion' in the sense of Pratt), even when δ is not very close to zero, for a given change in m, e will be unaffected by change in m given r_s.

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