Public Debt, Taxation, and Capital Intensiveness*

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In his "Principles of Political Economy and Taxation" [9], Ricardo cautioned that the deficit-financing of public expenditures sets back the growth of capital. The reference was to economies with rapid adjustment to "full-employment" equilibrium. While the Keynesians denied the rapid-equilibration assumption, the postwar reevaluation of monetary policy led to a modern restatement of Ricardo's doctrine. In several papers, of which Samuelson's [10] is probably the best known, it was argued that, given the level of government expenditures, a tax reduction would increase consumption and thus restrict investment if monetary policy is used compensatorily to maintain aggregate income and employment at their targeted levels. This is a statical proposition, good for each instant in time, given the currently available capital stock. But the current capital intensiveness is dependent upon the past history of taxes, so that an intertemporal model is required if we are to deduce that a permanently increased capital intensiveness will be brought about by a permanent decrease in public indebtedness per man.

Analyses of this question have been few. In his parable of saving under population overlap, Samuelson [11] showed that the social "contrivance" of government-issued money (unbacked by government-owned capital or other interest-paying assets) would tend permanently to increase the rate of interest (thus tending to cure his economy from any inefficient permanent overinvestment); but whether ordinary public debt would do as well was left unexplored. Modigliani [6] in his life-cycle model of a stationary economy, argued that by permanently adding a dollar to the public debt, the government would ultimately and permanently displace exactly one dollar's worth of capital from private portfolios. Diamond [2] synthesized these two models and showed that, under certain stability and uniqueness conditions, a permanent addition to the debt per head would produce a permanent reduction of capital per head—though not generally in an equal amount.

The present note shows that a government intent upon permanently increasing the economy's capital intensiveness through fiscal policy may find that as it succeeds the public debt per head has been increased and the necessary deficit per head has grown. Normally, however, there will be a "classical" range of capital-labor ratios in which an increase in long-run capital per head requires the government to pursue a policy that decreases the long-run public debt per head. These results depend only on the patent geometrical possibilities for the long-run consumption function, quite free of the model generating that function.

In a model in which the demand for consumption is a fixed fraction of disposable income, we show that there exists just one classical range and just one "anticlassical" range. Within the classical range, it is precisely at the Golden Rule capital–labor ratio—more generally, at the capital–labor ratio that maximizes sustainable private consumption—that a dollar of additional debt displaces exactly a dollar of capital. The reason is that it is just when consumption is maximal (across steady states) that one can, by the familiar envelope theorem, ignore the feedback effect of capital's displacement upon itself. As a corollary, it follows that, in this model, private wealth, defined as the sum of public debt and capital, is maximized at the Golden Rule capital–labor ratio. More generally, if the consumption function depends solely on private wealth and disposable income, then in the Golden Rule steady state, private wealth is maximized and a small increase of public debt would displace an equal dollar amount of capital. When in addition the consumption function depends upon the wage and interest rates, equal displacement and wealth maximization occur at the Schumpeterian zero-interest-rate steady state.

We then go on to discuss some welfare aspects of these behavioral relations. The point that an initial public debt is not a burden if it can be costlessly neutralized is reiterated. The absence of lump-sum taxes raises the possibility that the debt cannot be completely neutralized, because the available tax instruments may have substitution effects upon the allocation of time between work and leisure as well as between consumption and saving. The consequences of debt creation (or debt retirement) for future tax rates and corresponding future substitution effects must be considered, along with the current substitution effects of current tax rates, in the selection of the budgetary deficit program.

1. GOVERNMENT DEBT, SUSTAINABLE OUTPUT, AND CONSUMPTION

For ease of exposition, we begin with the simplest neoclassical model, showing later how the analysis can be extended to more general models.

Production follows the usual one-sector technology,

$$y = c+z = f(k),$$ (1.1)

where output per man, y, can be divided into consumption per man, $c \geq 0$, and investment per man, $z \geq 0$. At every instant, capital and labor are inelastically supplied, and the capital–labor ratio is denoted by k. If $n > 0$ is the constant rate of labor force growth, then the change in the capital–labor ratio is given by

$$k = z - nk,$$ (1.2)

ignoring capital depreciation.

The household demand for consumption is a fixed fraction, $0 < (1-s) < 1$, of private disposable income, which is here comprised of rewards to privately owned factors and government transfers less taxes. Let us suppose that the government, through central bank action, is able to keep the economy along an equilibrium (full-employment) path with zero inflation. Then private demand for consumption goods per man is given by

$$(1-s)[f(k)+\phi],$$

where ϕ denotes net government transfers per head. If government expenditure is zero, then ϕ is equal to δ, the per capita deficit, so that in momentary equilibrium

$$c = (1-s)[f(k)+\delta].$$ (1.3)

Government debt per head, denoted by Δ, therefore follows the simple law of motion

$$\Delta = \delta - n\Delta,$$ (1.4)

so that in balanced growth equilibrium $\Delta = \delta/n$.

It should be noted that the model has two leading interpretations. First, as in Diamond [2], the public debt could be thought to consist of demand loans held by households (somewhat like postal savings deposits but fixed in consumption units) which are perfect substitutes for capital and therefore pay a dividend rate, r, which is always equal to the return on capital, $f'(k)$. Second, the model could be considered to be the "reduced form" of a more complete model like that of Foley, Shell, and Sidrauski [3] in which the public holds three assets, capital, money (noninterest-bearing government debt), and short-term bonds (like postal savings deposits)—none of which is a perfect substitute for any other.[1] In this interpretation of the model, it must be assumed that the

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[1] For steady-state analysis only, one can admit bonds of any *finite* maturity, for then they will all be valued at par in steady states. Consols would raise valuation complications, though even these vanish as the steady state is approached.

central bank is able to hold the general price level constant by varying the debt–money ratio, through open-market purchases and sales, while the treasury controls the demand for consumption by its deficit policy.[2]
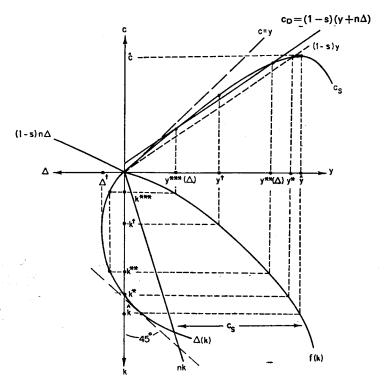

FIG. 1. Balanced-growth relations: sustainable per capita consumption supply, c_s, steady-state desired consumption, c_D, and steady-state per capita debt, $\Delta(k)$, as functions of the steady-state capital-labor ratio, k.

Balanced-growth ($\dot{k} = 0 = \dot{\Delta}$) solutions to the system (1.1)–(1.4) are described in Fig. 1. Output per head, $f(\cdot)$, and the nk-ray are plotted against the k-axis in the southeast quadrant. Sustainable per capita consumption supply, c_s, is equal to the difference between $f(k)$ and nk. Since $y = f(k)$, c_s can then be plotted against y in the northeast quadrant.

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[2] It may be noted that should there exist a discrepancy between the central bank's liabilities and its assets, net public indebtedness will differ *pro tanto* from the Δ of (1.4). But no such discrepancy can exist in steady states, at least not for $n > 0$ and price-level stationarity.

Under the usual regularity conditions in production,[3] c_S achieves a maximum at the Golden Rule capital–labor ratio, $\hat{k}$, where the rate of interest equals the rate of growth, $f'(\hat{k}) = n$. Since steady-state consumption is less than steady-state output, the c_S locus lies below the $c = y$ ray. c_S is zero when y is zero, rises to a maximum at the Golden Rule per-capita output, $\hat{y}$, and falls to zero at the maximum sustainable per-capita output, where $f(k) = nk$. Since $f(\cdot)$ is strictly concave in k, c_S is a strictly concave function of steady-state per-capita output. Because of the Inada condition $f'(0) = \infty$, as y becomes small, the c_S schedule approaches tangency to the $c = y$ ray.

In balanced growth, $\dot{\Delta} = 0$, so that $\delta = n\Delta$ and desired per-capita consumption is equal to $(1-s)(y+n\Delta)$. Hence, in the present model, the steady-state desired per-capita consumption locus is a straight line in the northeast quadrant that intersects the vertical axis at $(1-s)n\Delta$. We are now ready to study existence and uniqueness of balanced growth states, along with important propositions in comparative dynamics.

If in the steady state debt per head is zero ($\Delta = 0$), then there exists the unique (Solow) steady-state output per head, y^*, for in this case c_D is a ray from the origin. For s sufficiently small, $y^* \leq \hat{y}$ and development is intertemporally efficient. If, however, s is large, then $y^* > \hat{y}$ and development is intertemporally inefficient.[4]

If the government is a long-run creditor, then, for given $\Delta < 0$, steady-state output per head is uniquely determined. Again, for sufficiently small s, steady-state output per head is less than or equal to $\hat{y}$ and development is intertemporally efficient, while for larger s, steady-state $y > \hat{y}$ and development is intertemporally inefficient.

The case in which the government is a long-run debtor is more complicated. When $\Delta > 0$, the steady state is unique if and only if the c_D line is tangent to the c_S curve. Since $c_D = (1-s)(y+n\Delta)$,

$$(\partial c_D/\partial k) = (1-s)(\partial y/\partial k).$$

But $c_S = y-nk$, so that $(\partial c_S/\partial k) = (\partial y/\partial k)-n$. Therefore at the point of tangency, $y^\dagger$, we have that $sr = n$, where $r = f'(k)$ denotes the marginal product of capital. The unique debt per head consistent with the $y^\dagger$ steady state is denoted by $\Delta^\dagger$.

For $\Delta > \Delta^\dagger > 0$, c_D must everywhere exceed c_S, so no steady state is possible. Hence, $\Delta^\dagger$ is the maximum sustainable debt per head. For given Δ such that $\Delta^\dagger > \Delta > 0$, there exist exactly two steady-state per-

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[3] $f(k) > 0$, $f'(k) > 0$, $f''(k) < 0$ for $0 < k < \infty$, while $f(0) = 0$, $f(\infty) = \infty$, and $f'(0) = \infty$, $f'(\infty) = 0$.

[4] See Phelps [8].

capita outputs $y^{**}(\Delta)$ and $y^{***}(\Delta)$ with $y^* > y^{**}(\Delta) > y^\dagger > y^{***}(\Delta) > 0$.

Notice that government debt "matters." As Δ is increased, the c_D line is shifted upward. Therefore, in a steady-state equilibrium with positive debt per head, output per head is always less than in the Solow steady state ($\Delta = 0$). Similarly if $\Delta < 0$, output per head is always more than in the Solow ($\Delta = 0$) steady state, y^*.

If we restrict our attention to efficient steady states, where $y \leq \hat{y}$, then we know that the per-capita consumption is lower in the steady states with positive per-capita debt than in the steady state with zero per-capita debt. Similarly, steady state consumption is higher when $\Delta < 0$ than when $\Delta = 0$ as long as $y \leq \hat{y}$. It is easily seen from Fig. 1 that these propositions about steady-state per-capita consumption are reversed in the regimes for which $y > \hat{y}$.

Following previous authors, we ask the broader question: Is it in general true that a higher steady-state per-capita output must be accompanied by a lower steady-state per-capita government debt? We conclude from Fig. 1 that the answer is no. Notice that for each feasible y there exists exactly one steady-state Δ.[5] For $y > y^\dagger$, $d\Delta/dy < 0$, since $y^{**}(\Delta)$ falls as the c_D line is shifted upward. But for $y < y^\dagger$, $d\Delta/dy > 0$, since $y^{***}(\Delta)$ rises as the c_D line is shifted upward.[6] We summarize these results in the first proposition.

PROPOSITION 1. *Across steady states, there is a classical range where* $d\Delta/dk < 0$ *and an anticlassical range where* $d\Delta/dk > 0$, *with*

$$\text{sign} \, (d\Delta/dk) = \text{sign} \, (sr-n).$$

[The surprising decline of debt per head as capital per head falls, in the anticlassical (high-interest-rate) range, does not imply that debt per unit of output also falls in that range. Indeed, it is easy to show that the debt-output ratio is monotone-decreasing in the capital–labor ratio. Corresponding to every steady-state debt-output ratio is a deficit–output ratio and hence some fixed multiple between output per head and per capita disposable income. Hence, in this case, c_D, the long-run consump-

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[5] Although the steady-state y is not a single-valued function of steady-state Δ, Δ is a single-valued function of y defined over the interval $[0, \bar{y}]$, where $\bar{y}$ is the maximum sustainable output per man.

[6] It might be argued that the anticlassical high-interest-rate regime has limited empirical relevance. If the growth rate is even as low as 3 percent and saving is equal to 10 percent of income, then $d\Delta/dk > 0$ only when the government is planning for a long-run return on investment in excess of 30 percent. We do not know, however, whether this anticlassical regime would seem more or less remote from observed and contemplatable capital intensities in more complex models.

tion function, starts at the origin. As the debt–output ratio is increased from its smallest (negative) sustainable value, the steady-state capital–labor ratio decreases, tending asymptotically to zero because of our assumption that $f'(0) = \infty$.]

Remember that if $c(y)$ is steady-state consumption per man, then $c(y)$ is at a maximum at $\hat{y}$ and sign $(dc/dy) = $ sign $(\hat{y} - y)$. From Proposition 1, we can now deduce the next result.

PROPOSITION 2. *Across steady states, per capita consumption is positively associated with the debt both in the "anticlassical" range and in the inefficient portion of the classical range; that is,*

$$\frac{dc}{d\Delta} \begin{cases} > 0 & \text{if } r > n/s, \\ \text{undefined} & \text{if } r = n/s, \\ < 0 & \text{if } n < r < n/s, \\ = 0 & \text{if } r = n, \\ > 0 & \text{if } 0 < r < n. \end{cases}$$

For purposes of construction, we draw the $(1-s)n\Delta$ ray in the northwest quadrant of Fig. 1. The intercept of c_D with the vertical axis is equal to $(1-s)n\Delta$. Therefore, by projecting these c_D intercepts through the $(1-s)n\Delta$ ray, we are able to derive the steady-state relation between capital per head and debt per head. This relation is described by the $\Delta(k)$ locus in the southwest quadrant. $\Delta(k)$ is zero when k is zero, rises to a maximum $\Delta^\dagger$ when $k = k^\dagger$, and falls to zero when $k = k^*$ (the Solow steady-state capital–labor ratio). For $k > k^*$, steady-state Δ is negative.

From (1.3), steady-state per capita consumption, c, is equal to $(1-s)[f(k)+n\Delta]$, since in balanced growth $\delta = n\Delta$. Therefore,

$$\left(\frac{1-s}{s}\right) n(k+\Delta) = f(k) - nk, \tag{1.5}$$

because $c = f(k) - nk$ in steady states. Differentiating (1.5) with respect to k yields

$$\left(\frac{1-s}{s}\right) n\left(1 + \frac{d\Delta}{dk}\right) = f'(k) - n. \tag{1.6}$$

Remember that at $\hat{k}$, the Golden Rule capital–labor ratio, the rate of interest, $f'(\hat{k})$, is equal to the growth rate, n. We conclude from (1.6), that $(d\Delta/dk) = -1$ if and only if $k = \hat{k}$. Therefore, in Fig. 1, $\Delta(k)$ is tangent to the 45° line at $\hat{k}$. We summarize this result in the next proposition:

PROPOSITION 3. *A dollar of government debt permanently displaces exactly one dollar of private capital (if it displaces capital at all) only in the Golden Rule steady-state, and not elsewhere.*

That the public debt permanently displaces an equal dollar amount of real capital from the portfolios of households was suggested by Modigliani [6]. It is of interest, therefore, that for our model, Modigliani's conclusion holds only in the neighborhood of the Golden Rule steady state. Proposition 3 is actually a simple instance of the familiar envelope theorem and consequently applies to a wider class of models. In more general models, steady state desired per capita consumption depends upon wealth per man, $w = k + \Delta$, disposable income per man, $h = y + n\Delta$, the wage rate, ω, and the interest rate, r. In balanced growth, therefore,

$$c_S(k) - c_D(w, h, \omega, r) = 0.$$

But in steady-state equilibrium,

$$h = f(k) + n\Delta = c_S(k) + nk + n\Delta = c_S(k) + nw.$$

Thus,

$$c_S(k) - \psi(w, y) = 0,$$

since y uniquely determines ω and r. Implicit differentiation in the above equation yields

$$\frac{d\Delta}{dk} = \frac{(dc_S/dk) - \psi_1(\partial w/\partial k) - \psi_2(dy/dk)}{\psi_1(\partial w/\partial \Delta)}.$$

But since $\partial w/\partial k = 1 = \partial w/\partial \Delta$ and $r = dy/dk$,

$$\frac{d\Delta}{dk} = \frac{(dc_S/dk) - \psi_1 - r\psi_2}{\psi_1}.$$

If c_D depends only on income and wealth, i.e. if $\psi_2 \equiv 0$, then $d\Delta/dk = -1$ if and only if $k = \hat{k}$. That is, when c_S is maximal, the first-order change in Δ, *mutatis mutandis* (c_S allowed to vary) owing to a change in k is equal to the first-order change in Δ *ceteris paribus* (c_S constant) owing to a change in k. Most generally, equal displacement occurs if and only if

$$r - n - r\psi_2 = 0.$$

Thus in the Schumpeterian (zero-interest-rate) steady state, with no population growth, $d\Delta/dk = -1$, since $n = 0 = r$.

In Modigliani's model [6], the "desired" ratio of wealth to disposable income is constant, and disposable income equals national income in any stationary-state. But even in that special case, the feedback of capital displacement upon income is present for positive interest rates so that it is only in the Schumpeterian (Golden Rule) stationary state that exact displacement is ensured.

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Differentiating (1.6) with respect to k yields

$$\left(\frac{1-s}{s}\right) n \left(\frac{d^2\Delta}{dk^2}\right) = f''(k) < 0. \tag{1.7}$$

Therefore, as shown in Fig. 1, $\Delta(k)$ is a concave function of k. Then, as corollary to Proposition 3, we have for the simple model described by (1.1)–(1.4) that private wealth per man, $w = \Delta + k$, is maximized if and only if consumption is maximized, i.e., when $k = \hat{k}$.[7] Furthermore, this result holds for any consumption function which depends solely on private wealth and disposable income, since when $\psi_2 = 0$, $dw/dk = 0$ i.e., $d\Delta/dk = -1$) if and only if $dc_S/dk = 0$. Even when c_D depends on y and r, w is maximized in the Schumpeterian state, where $r = 0 = n$.

Some generalizations of the model described in Fig. 1 should be mentioned. If we introduce a fixed amount of government expenditure per man, both the c_S and c_D schedules are affected. The former curve is displaced downward by the amount of the public outlay per head. The variable ϕ now replaces δ, where $\phi = \delta - g$, g being the government expenditure per capita. Because $(1-s) < 1$, the balanced-budget theorem operates: The wedge of government expenditure reduces per-capita consumption supply, c_S, by more than demand, c_D. In the classical range, where the slope of c_S is less than the slope of c_D, the effect is to reduce steady-state investment per man, output per man, and capital intensiveness; the opposite results occur in the anticlassical range, where the slope of c_S is greater than the slope of c_D. Aside from the fact that a range of small capital–labor ratios are not sustainable with fixed $g > 0$, the analysis of the effects on k of a change in Δ are not essentially affected.

If per capita government expenditure is made a function of output per head, still different results occur. If $g = \gamma y$ with γ a fixed fraction, then the slope of the c_D function is $(1-s)(1-\gamma)$. The c_S curve will also have a smaller slope at every y since the government "takes its cut" of output left over after the capital formation necessary to maintain the capital–labor ratio. This means that the private-consumption-maximizing point, where one gives no weight to public expenditures in measuring total consumption, occurs at an interest rate greater than the growth rate, $r > n$ and $y < \hat{y}$. It is at this point, where the c_S curve has a zero slope, that a dollar of additional debt per head exactly displaces one dollar of capital per head. The critical interest rate is given by

$$r(1-\gamma) - n = 0, \tag{1.8}$$

and the point $y^\dagger$, separating the classical and anticlassical ranges, occurs

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[7] See also Levhari and Patinkin [4], especially pp. 746–747.

where the interest rate is given by

$$r = \frac{n}{s(1-\gamma)+\gamma}.$$ (1.9)

In a life-cycle model, with or without bequests, the steady-state desired per capita consumption locus will not generally be linear. Imagine confronting each household with a configuration of a wage rate, an interest rate, and government transfers per head which are constant over time. There will correspond a steady-state, desired-wealth level, $w_D = (k+\Delta)_D$, and a steady-state per capita desired-consumption level, c_D. Since the interest rate and the wage rate are monotonic functions of output per man, y, such a model implies that

$$c_D = C(y, \Delta; n),$$

which need not, of course, be linear in y or in $\Delta(= \delta/n)$. Nonlinearity raises the possibility of multiple intersections of the c_D locus with the c_S curve. Then the functional relationship between Δ and k will exhibit a relative maximum or minimum corresponding to each tangency of c_S with c_D as the latter is shifted as a consequence of varying Δ. There will be a paradoxical anticlassical range corresponding to each relative maximum.

In the life-cycle model, c_D is ordinarily a monotonically increasing function of y. It is also plausible to suppose that c_D shifts upward as Δ is increased and that for positive Δ the c_D locus has a positive intercept with the vertical axis.[8] Since c_S is a concave function starting from the origin, if a steady-state with $\Delta > 0$ exists, $c_D = c_S$, then there must be at least one steady-state in which the slope of c_S is greater than or equal to the slope of c_D. If at the steady-state, the slope of c_S is strictly greater than the slope of c_D, then there must exist at least one other steady-state for which $(\partial c_D/\partial y) > (\partial c_S/\partial y)$. Thus, with $\Delta > 0$ the steady-state in the life-cycle model will only be unique in the singular case of a unique tangency of c_S with c_D.[9] If a non-tangency steady-state exists for given $\Delta > 0$, there must be an anticlassical range of capital–labor ratios for which $d\Delta/dk > 0$.

Another generalization is to allow for many capital goods. Then the c_S curve may have many local maxima and minima, though the Golden Rule point continues to be the global maximum. This too raises the

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[8] An exception is the model of Bailey [1], where households (with bequest motives) presently value the discounted stream of taxes needed to service the public debt as exactly equal to the value of the outstanding interest-bearing public debt.

[9] This proposition depends upon the fact that c_S achieves a maximum. If we relax our assumptions about the regularity of the production function, $f(k)$, different results can occur. If c_S is monotonically increasing, then the steady-state may be unique for given $\Delta > 0$, and $d\Delta/dk$ would be positive for any $k > 0$.

possibility of multiple intersections with the c_D locus. Therefore, there may be multiple anticlassical ranges on this account as well. Further, even when c_D depends solely on wealth per head and disposable income per head, there may be many points of "equal displacement," each one yielding a relative maximum or minimum of total dollar wealth per head as a function of the dollar value of capital per head (since each one corresponds to a locally flat c_S locus).

2. DYNAMIC ANALYSIS

In the previous section, we studied steady-state behavior and derived certain propositions in comparative dynamics—in particular, that across steady-state sign $(d\Delta/dk) = \text{sign } (sr-n)$. In this section, we turn to the full dynamic analysis. This is important because, as we shall see, a certain stability analysis and a closely related assumption about uniqueness of the balanced growth state seems to be fundamental to Diamond's [2] claim that $d\Delta/dk < 0$ across steady-states. Further, such a dynamic analysis is crucial for understanding the relevance of the result.

Remember that in the one-sector, constant-saving-fraction model when long-run debt per man is chosen to be positive but less than the maximum value $\Delta^\dagger$, long-run output per man is not unique. At the lower output per man $y^{***}(\Delta)$, the slope of c_S exceeds the slope of c_D. Since, in Fig. 1, c_D shifts upward as Δ is increased, $dy^{***}/d\Delta$ and $dk^{***}/d\Delta$ are positive. The question immediately arises: If c_S has greater slope than c_D, is not the y^{***} equilibrium unstable? The answer is that this need only be true if we limit the government to the pursuit of policies with constant deficits per man. This point will require further analysis.

From Eqs. (1.1)–(1.3), we can derive the equation for capital accumulation,

$$k = sf(k) - [(1-s)\delta + nk]. \tag{2.1}$$

In Eq. (2.1), the deficit per man, δ, can be set by the government at each instant to bring forth any desired investment consistent with existing endowments of capital and labor. On this postulate of full fiscal effectiveness, the government can achieve any technologically feasible time-path of consumption and capital accumulation.

For example, the government might follow some rule that makes planned per capita consumption, c^0, a function of the inherited capital-labor ratio.[10] The desired deficit per man, δ^0, can then be calculated

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[10] In many contemporary planning models, a Ramsey-like optimal economic growth policy implies that planned consumption per head will be a uniquely determined increasing function of k. The implications of a Ramsey-optimal economic growth policy for

as a function of the capital–labor ratio, since from (1.3)

$$c^0(k) = (1-s)[f(k)+\delta^0(k)].$$

Given $c^0(k)$, we can calculate $\delta^0(k)$, and thus $z^0(k)$ and $k^0(k)$.

The process of capital accumulation can be more fully analyzed in Fig. 2. *For the moment*, we assume, like Diamond [2], that the government
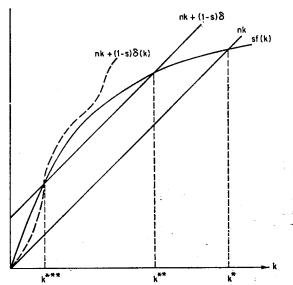

FIG. 2. Dynamic analysis.

holds the deficit per man, δ, constant throughout the adjustment path. For $\delta = 0$, $\dot{k}$ is the difference between the $sf(k)$ curve and the nk-ray. As Solow showed, k^* is unique and is globally stable, i.e.,

$$\text{sign } \dot{k} = \text{sign }(k^*-k) \quad \text{for } k > 0.$$

For fixed $\delta < 0$, the $[nk+(1-s)\delta]$ line lies below the nk-ray. Again, with δ held constant the unique long-run balanced growth equilibrium is globally stable.

For fixed $\delta > 0$, the story is more complicated. In the *neighborhood* of k^{**}, sign $\dot{k} = $ sign $(k^{**}-k)$, yielding that k^{**} is *locally* stable when the government holds δ constant. In the neighborhood of k^{***}, however, sign $\dot{k} = $ sign $(k-k^{***})$, so that k^{***} is unstable when δ is held constant.

fiscal and monetary policy are examined by Foley, Shell, and Sidrauski [3]. For a two-sector mixed economy with optimal fiscal and monetary policy, they show that in balanced growth, sign $(d\Delta/dp) = $ sign $(-d\Delta/dk) = $ sign $(n - sr)$, where $p > 0$ is the government's pure (subjective) rate of time discount for per capita consumption.

If the government maintains a constant deficit per head, then when the economy is disturbed from k^{***}, it will not return.

Diamond [2] excludes from his comparative-dynamics theorem steady states such as k^{***} which are dynamically unstable under fixed δ regimes. In the model described by Eqs. (1.1)–(1.4), $d\Delta/dk \geq 0$ if and only if the corresponding steady-state is unstable when δ is held constant. The k^{***} steady-state would, however, be globally stable if the government chose δ to be small when k is small and δ to be large when k is large. Such a rule, in which the deficit per man, $\delta(k)$, depends upon the capital–labor ratio, is described in Fig. 2. The dashed curve represents $[nk + (1-s)\delta(k)]$. For $k < k^{***}$,

$$sf(k) > [nk + (1-s)\delta(k)],$$

so that $\dot{k} > 0$. Similarly, for $k > k^{***}$,

$$sf(k) < [nk + (1-s)\delta(k)],$$

so that $\dot{k} < 0$. Hence, we have demonstrated how the government can choose a policy so that independent of initial endowments the economy ultimately tends to k^{***}.

For the simple constant-saving-fraction model, a steady-state equilibrium is locally stable for fixed δ *if and only if* $\partial c_D/\partial y > \partial c_S/\partial y$. In the life-cycle model, however, a steady-state with the slope of c_S greater than the slope of c_D (and thus $d\Delta/dy > 0$) is not necessarily unstable *even under constant δ regimes*. This is because the "momentary consumption functions" relating current per capita consumption to current income per head may be sufficiently steeper than the long-run c_D locus. Even though the long-run c_D locus is flatter than the long-run c_S locus at equilibrium, the "momentary consumption function" may be steep enough to ensure the stability of a constant-deficit-per-man policy.[11]

The reader will now readily see through our paradox of "capital deepening through fiscal ease"—in the anticlassical range. In the transition to a greater capital per head, the government must assuredly reduce at least temporarily the algebraic deficit, raising taxes and reducing consumption at first. But as the capital–labor ratio rises, the increase in sustainable consumption is so great, in the anticlassical range, relative

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[11] Such cases do not figure in the general analysis presented by Diamond [2]. He excluded them from the text, apparently on the ground that he wished to deal primarily with the case in which the curves describing short-run interest-wage determination intersect in such a way that a Walrasian *tatonnement* process would be stable (see [2], p. 1132). In an appendix, however, he showed that the opposite assumption, allowing for a stable Marshallian adjustment process, in no way interferes with the convergence of the economy to its golden age equilibrium. He noted that in this case $dk/d\Delta > 0$ across golden ages.

to the increase in consumption demand which would occur if the original
deficit per head were restored that even the *original* deficit is too small
to establish equilibrium at the higher capital–labor ratio: A higher
deficit is required. This is faintly reminiscent of the doctrine of secular
stagnation with this important difference: In the present model, monetary
policy can maintain full employment despite the lower yield on capital,
but deficit spending is needed to keep the superfluity of private thrift from
leading to still further capital deepening.

WELFARE ASPECTS

Much of the debate on the "burden of the debt" is beclouded by
semantic difficulties. Many writers have termed the public debt "burden-
some" if long-run consumption per man is decreased when government
debt per man is increased. Diamond [2] showed that in this sense the
debt is not a burden for economies pursuing intertemporally inefficient
development programs. For such cases, long-run $d\Delta/dk < 0$ but long-run
$d\Delta/dc > 0$. We have shown that in addition to the inefficient, low-interest-
rate range, there is also a high-interest-rate range within the efficient range
for which steady-state $d\Delta/dc > 0$ and $d\Delta/dk > 0$.

Even in that part of the "classical" range that is short of the Golden
Rule point, namely, $n < r < n/s$, so that $d\Delta/dc < 0$, the term "burden"
is unfortunate for prejudicing fiscal policies which increase the debt.
At any moment, a government fully aware of the consequences of its
actions might choose an easy fiscal policy coupled with a tight monetary
policy, i.e., elect to finance a given government expenditure partly through
a deficit rather than by taxation. Such policies are not necessarily irrational
merely because they promote current consumption at the expense of
future consumption: The present benefit may be thought to outweigh
the future loss from that policy.

But the central objection to the term "burden" is that the inherited
stock of debt, as distinct from increases in it, cannot be a burden if, as
in the context of the model described by Eqs. (1.1)–(1.4), the government
can neutralize the allocative and distributive influences of that debt by
means of suitable taxation. In choosing an optimal fiscal policy program
for the (possibly infinite) planning period, the government must, at least
implicitly, rank attainable growth paths. Suppose, for example, the
government's criterion functional depends only on the time-path of per
capita consumption. Then maximal attainable welfare at time t, W,
depends only upon inherited endowments at that time:

$$W[k(t), \Delta(t)].$$

In our model, future consumption possibilities are enhanced when the

inherited capital–labor ratio is higher, so that $\partial W/\partial k > 0$. If, as assumed in the previous sections, fiscal policy can achieve any technologically feasible consumption path, no matter what the size of the inherited debt, then $\partial W/\partial \Delta = 0$ for all Δ.

The proposition just enunciated requires no assumption about the availability of lump-sum taxes. If only the per capita consumption sequence figures in the social utility function, then the possibility that the fiscal instrument employed to control that variable may have side effects on other allocations is of no welfare significance.

If, in contrast, labor supply is not invariant and if the time-path of per capita leisure, as well as per capita consumption figures in the objective functional then it will not *generally* be possible, with just one fiscal instrument, to guide these two variables along the best technologically feasible path. Singular cases exist, the most obvious being that in which, for every per-capita consumption rate the government engineers, each household is automatically led by the market to choose the social-utility maximizing amount of leisure because, in a certain sense, social preferences between goods and leisure correspond to individual private preferences and because the kind of tax in use to control consumption, like the essentially fictitious "lump-sum" tax, does not "distort" the labor-leisure decision. When social and private preferences are alike (as between goods and leisure) but the kinds of tax in use for controlling consumption are "distorting," like the income tax, the technologically feasible optimum is not generally attainable. These taxes will distort, through the substitution effect, the leisure-goods allocation, as well as private thrift.

When the technologically feasible optimum is not generally attainable by virtue of an insufficiency or imperfectness in the fiscal instruments, it is also the case that the initial public debt cannot generally be exactly neutralized. The usual argument is that the additional taxes necessary to offset the wealth and income effects upon consumption demand will have substitution effects upon the effort-leisure choice.[12] But one cannot exclude the possibility that, by chance, the "neutralizing" increment in tax rates will serve to reduce consumption and leisure demand in just the right proportions so as to make reattainable the *status quo ante debitum*.[13]

Despite the analytical complexities of the matter, the size of the "tax rate" remains of some interest as a cause of resource misalignment. If we restrict our attention to that interpretation of our model in which short-term government bonds are perfect substitutes for capital and there is no money among the government liabilities, then there is a relationship

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[12] See Meade [5].

[13] See Phelps [7].

between τ, the tax rate expressed in terms of taxable income, the latter defined to include the interest on the debt, and debt per head,

$$\tau = \frac{\gamma y + (r-n)\Delta}{y + r\Delta},$$

where $r = f'(k)$. On the one hand, the interest on the debt increases the tax rate necessary to yield the deficit that corresponds to the debt per head, Δ. This effect is attenuated by the accompanying enlargement of the tax base. On the other hand, interest apart, the larger the debt per head the larger the deficit must be, and hence the smaller the required tax rate, in order that the debt keep pace with the growing population.

We can study the derivative, $d\tau/d\Delta$, always remembering k is not a single-valued function of Δ, so that the derivative must be thought of as $(d\tau/dk)/(d\Delta/dk)$. The relation is

$$(y+r\Delta)^2 \frac{d\tau}{d\Delta} = [(1-\gamma)r - n](y - \Delta y') + \Delta r'[(1-\gamma)y + n\Delta],$$

where primes denote differentiation with respect to Δ. In the classical range, where the derivatives y' and r' are negative and positive, respectively, we see that at least for nonnegative Δ, the tax rate is increasing with the debt up to the modified Golden Rule point where $(1-\gamma)r = n$; at sufficiently larger capital–labor ratios, there appears to be some ambiguity, since reaching these low-interest rates may require negative debt. In the high-interest anticlassical range, $y' > 0$ and $r' < 0$. For large enough, positive Δ, therefore, the sign of the derivative is again in question.

We have uncovered and sought to explain some of the surprising relationships that exist among capital per head, public indebtedness per head, and the income tax rate in simple mixed-economy models in which the government may use fiscal instruments to influence household consumption demand and thus to control the growth-path of the economy. In such models, where long-run, steady-state behavioral loci may be misleading for stability analysis, and where stability analysis is itself beside the point when the government is in effect altering the private response to data changes in order to secure some desired result, there will generally exist some surprising anticlassical relations among these three variables—debt, taxes, and capital. A fiscal policy designed to reduce capital intensiveness for a near-term gain of consumption may end up permanently reducing per capita public indebtedness, and, whether or not it decreases the debt, it may also, though it need not, reduce the income tax rate.

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Erratum

Vol. **1**, No. 3 (1969), in the article "Public Debt, Taxation, and Capital Intensiveness," by E. S. Phelps and K. Shell, p. 339:

Howard A. Chernick, University of Pennsylvania, has pointed out an error in Eq. (1.9). It should read:

$$r = \frac{n}{s(1 - \gamma)}.$$