Price-Level Volatility and Optimal Taxation

Guido Cozzi
University of St.Gallen

Aditya Goenka
University of Birmingham

Minwook Kang
Nanyang Technological University

Karl Shell*
Cornell University

September 9, 2015

Abstract

We analyze a very simple economy in which taxes (employed purely for income redistribution) are denominated in money units (say, dollars). Volatility of the price-level is sunspot-driven. Some agents cannot participate in the market for hedging against fluctuations in the price level. The tax authority chooses money taxes to maximize Benthamite welfare, i.e., the sum of expected utilities. Optimization entails leveling the expected utilities among the group of consumers who have access to the hedging market. The money-taxation regime is compared to a commodity-taxation regime in which transfers suffer from (iceberg) spoilage. In the commodity-tax regime, optimization implies that all taxed consumers receive the same utility and that all subsidized consumers receive the same utility. The cost of money taxation is in volatility, while the cost of commodity taxation is the partial spoilage of commodity in the tax-transfer process.

1 Introduction

Finance plays a very important, and largely positive, role in advanced economies, but it can contribute to excess economic volatility. We build a simple model of taxation in terms of fiat money, our financial instrument. Price-level volatility is driven by sunspots. There is an information friction: Only some agents

* Corresponding author: Karl Shell, 402 Uris Hall, Cornell University, Ithaca, NY 14853-7601 USA, email: karl.shell@cornell.edu

Goenka would like to thank NUS Grant FY2012-FRC4-003 for research support.

can hedge against price-level volatility. Others cannot. The friction can be interpreted as in Cass and Shell (1983) as a restriction on market participation because (for example) some individuals are not alive while the security market is open. Another interpretation is that this is a special case of asymmetric (or correlated) information.[1]

The tax authority is assumed to choose money taxes that maximize the sum of expected utilities. If there were no frictions, price-level volatility would not affect utilities or welfare. Otherwise, welfare is strictly decreasing in volatility. Our present model is an extension from *exogenous* money taxation to endogenous taxation. See Bhattacharya, Guzman, and Shell (1998) and Cozzi, Goenka, Kang, and Shell (2015). In these two exogenous tax papers, the tax authority's response to volatility is absent, since in these papers taxes are pre-determined.

We compare the financial money-taxation economy with the non-financial commodity taxation economy. The welfare cost of taxation in the financial economy is purely from volatility. The non-financial economy does not suffer from volatility,[2] but it does suffer from iceberg-style spoilage of net tax commodity transfers. We show that, for the commodity taxation case, optimization of welfare entails equalization of the utilities of the taxed consumers and equalization of the utilities of the subsidized consumers. We show, in terms of the volatility rate and the spoilage rate, which regime is chosen by the tax authority.

This is the first in a two-paper series on endogenous money taxation. The present paper is on optimal taxation. The next paper is on voting.

2 The Economy

There is 1 period and 1 consumption good (say, chocolate). There are 3 consumers, $h = 1, 2, 3$. The consumption of Mr. h is $x_h > 0$ and his endowment is $\omega_h > 0$. The consumers have identical logarithmic preferences given by

[1] See Aumann, Peck and Shell (1985), Aumann (1987), and Peck and Shell (1991).

[2] Goenka (1994) shows that an economy with real subsidies is immune to excess volatility, while one where the subsidies are denominated in value is susceptible to it.

the utility functions:

$$u_h(x_h) = \log(x_h) \quad \text{for} \quad h = 1, 2, 3.$$

These preferences (or, more generally, CRRA identical preferences) ensure that equilibrium is unique. We introduce sunspots (or, extrinsic uncertainty). There are two extrinsic states of nature $s = \alpha, \beta$, that occur with probabilities $\pi(\alpha), \pi(\beta), 0 < \pi(\alpha) < 1, \pi(\beta) = 1 - \pi(\alpha)$. We assume that Mr. h maximizes his expected expected utility

$$V_h = \pi(\alpha) \log(x_h(\alpha)) + \pi(\beta) \log(x_h(\beta)) \quad \text{for } h = 1, 2, 3.$$

The social policy instruments are lump-sum taxes $\tau = (\tau_1, \tau_2, \tau_3)$ denominated in units of money, say dollars. Each individual's tax is independent of the state of nature, i.e., $\tau_h(\alpha) = \tau_h(\beta) = \tau_h$ for $h = 1, 2, 3$. If τ_h is negative, $Mr.\ h$ is subsidized. If τ_h is zero, then he is neither taxed nor subsidized. The tax and transfer plan is balanced, i.e., $\tau_1 + \tau_2 + \tau_3 = 0$, else the goods price of money is zero.[3]

Let $p(s)$ be the ex-ante (accounting) price of the good delivered in state $s = \alpha, \beta$ and $p^m(s)$ be ex-ante (accounting) price of money delivered in state s. Then $P^m(s) = p(s)/p^m(s)$ is the chocolate price of money in s, while $1/P^m(s)$ is the money price of chocolate in s, or the general price level in s. The set of equilibria is typically very large, but we focus on a sub-set in which volatilities can be ranked. We measure volatility by the mean-preserving spread parameter σ defined by

$$P^m(\alpha) = P^m - \frac{\sigma}{\pi(\alpha)}$$

$$P^m(\beta) = P^m + \frac{\sigma}{\pi(\beta)}$$

where P^m is the non-sunspot equilibrium chocolate price of dollars and σ belongs to $[0, \pi(\alpha) P^m)$. When $\sigma = 0$, the equilibrium allocations are not affected by sunspots (a non-sunspots economy). When $\sigma > 0$, the economy

[3] See Balasko and Shell (1983).

is a proper sunspots economy. State α is the inflationary state: a dollar buys less chocolate in state α than in state β. State β is the deflationary state: a dollar buys more chocolate in β than in α.

3 Money Taxation and Social Welfare

The social welfare function W is the sum of the individual expected utilities. The tax authority chooses the tax τ to maximize welfare $V_1 + V_2 + V_3$. Define the maximized value of welfare by

$$W = \max_{\tau} V_1 + V_2 + V_3.$$

Figure 1 is the time-line.[4]

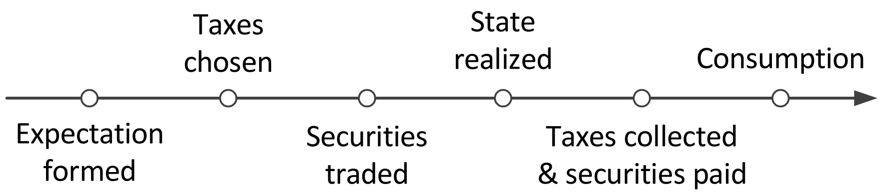


Figure 1: The time line

There are three basic cases based on the pattern of the asset market restrictions: (U) Unrestricted security market participation, allowing for perfect risk-sharing among the 3 consumers, (I) Incomplete securities-market participation allowing for risk-sharing between 2 of the consumers but not the third, and (R) Fully restricted securities-market participation, in which none of the consumers can hedge against price-level fluctuations. Denote $W(U), W(I)$ and $W(R)$ as social welfare under perfect risk-sharing market, under partially restricted market and under fully restricted market, respectively.

[4] We work in the traditional framework of economic policy formulation where consumers form price expectations, the policy maker then chooses the tax policy, and given these expectations an equilibrium outcome is realized. In equilibrium, the price expectations of consumers must be consistent with the equilibrium outcome: rational expectations must hold.

In the case of perfect risk-sharing, sunspots do not matter and the first-best social welfare is achieved. The most interesting case is when some consumers are restricted and others are not: The case of Incomplete Participation I. Consider, for example, the case in which Mr 1 and Mr 2 have access to the security market and Mr. 3 does not:[5]

The problem of restricted consumer 3 is simple. He chooses $x_3(s) > 0$ to

$$\text{maximize } \log\left(x_3(s)\right)$$

subject to

$$p(s)x_3(s) = p(s)\omega_3 - p^m(s)\tau_3$$

for $s = \alpha, \beta$.

Define the tax-adjusted endowment $\widetilde{\omega}_h\left(s\right) = \omega_h - P^m(s)\tau_h$. Then, *Mr* 3's budget constraints reduces to

$$x_3(s) = \widetilde{\omega}_3\left(s\right)$$

for $s = \alpha, \beta$. Mr 3 is passive: he consumes his tax-adjusted endowment in each state.

Mr 1 and Mr 2 trade in the securities market and the spot market. Each faces a single budget constraint. Mr h's problem is to choose $(x_h(\alpha), x_h(\beta)) > 0$ to

$$\text{maximize } V_h$$

subject to

$$p(\alpha)x_h(\alpha) + p(\beta)x_h(\beta) = \left(p(\alpha) + p(\beta)\right)\omega_h - \left(p^m(\alpha) + p^m(\beta)\right)\tau_h$$

for $h = 1, 2$. From the first-order conditions, we have

$$\frac{p(\beta)}{p(\alpha)} = \frac{\pi\left(\beta\right)x_1\left(\alpha\right)}{\pi\left(\alpha\right)x_1\left(\beta\right)} = \frac{\pi\left(\beta\right)x_2\left(\alpha\right)}{\pi\left(\alpha\right)x_2\left(\beta\right)}. \tag{1}$$

[5] The situation where only one consumer has access to security markets is not interesting as there is no counterpart to trade securities with.

Market clearing implies

$$x_1(s) + x_2(s) + x_3(s) = \omega_1(s) + \omega_2(s) + \omega_3(s)$$

or simply

$$x_1(s) + x_2(s) + x_3(s) = \widetilde{\omega}_1(s) + \widetilde{\omega}_2(s) + \widetilde{\omega}_3(s) \tag{2}$$

for $s = \alpha, \beta$. But $x_3(s) = \widetilde{\omega}_3(s)$, so we have

$$x_1(s) + x_2(s) = \widetilde{\omega}_1(s) + \widetilde{\omega}_2(s) \quad \text{for } s = \alpha, \beta. \tag{3}$$

Equation (3) defines the relevant tax-adjusted Edgeworth box, which is typically a proper rectangular, the indication that sunspots will matter in equilibrium.[6]

4 Social Welfare and Restricted Market Participation

The first-best value of social welfare W is

$$3 \log \frac{\omega_1 + \omega_2 + \omega_3}{3}.$$

Welfare will be no smaller than its value in autarky,

$$\log \omega_1 \omega_2 \omega_3.$$

The following proposition shows that with strictly positive price-level volatility, asset market restrictions (i.e., the information frictions) negatively affect the social welfare:

Proposition 1 *If $\sigma > 0$, we have*

$$W(U) \geq W(I) \geq W(R).$$

[6] See Cass and Shell (1983, p. 212 or Section V).

Proof. Proposition 1 can be proven by Lemma 1, Proposition 2 and Lemma 2. ■

Proposition 1 indicates that as the asset market becomes more restricted, the social welfare declines.

Lemma 1 $W(U) = 3\log\frac{\omega_1+\omega_2+\omega_3}{3}$

Proof. When the 3 consumers do perfect risk sharing, $p(\alpha)$ and $p(\beta)$ are invariant in σ:

$$\frac{p(\beta)}{p(\alpha)} = \frac{\pi(\beta)}{\pi(\alpha)}.$$

Each consumer chooses $x_h(\alpha) = x_h(\beta)$, because we have

$$\frac{x_h(\alpha)}{x_h(\beta)} = \frac{p(\beta)/\pi(\beta)}{p(\alpha)/\pi(\alpha)} = 1.$$

With $x_h(\alpha) = x_h(\beta)$, the equilibrium V_h can be expressed as

$$V_h = \log\{\omega_h - (P^m(\alpha) + P^m(\beta))\,\tau_h\}$$

and social welfare can be expressed as

$$W(U) = \max_{\tau_1,\tau_2,\tau_3} \sum_{h\in H} \log\{\omega_h - (P^m(\alpha) + P^m(\beta))\,\tau_h\}$$

$$\text{subject to } \tau_1 + \tau_2 + \tau_3 = 0.$$

By the first order conditions, we have

$$\frac{-(P^m(\alpha)+P^m(\beta))}{\omega_1 - (P^m(\alpha)+P^m(\beta))\,\tau_1} = \frac{-(P^m(\alpha)+P^m(\beta))}{\omega_3 - (P^m(\alpha)+P^m(\beta))\,\tau_2} = \frac{-(P^m(\alpha)+P^m(\beta))}{\omega_3 - (P^m(\alpha)+P^m(\beta))\,\tau_3},$$

which implies that

$$x_1(\alpha) = x_1(\beta) = x_2(\alpha) = x_2(\beta) = x_3(\alpha) = x_3(\beta) = \frac{\omega_1+\omega_2+\omega_3}{3}.$$

Therefore, we have

$$W(U) = 3\log\frac{\omega_1+\omega_2+\omega_3}{3}.$$

■

Proposition 2 *In the partially restricted market (I), if Mr h and Mr h' trade in the securities market to share risk, we have $V_h = V_{h'}$.*

Proof. Without any loss of generality, let $h = 1$ and $h' = 2$. Because Mr 3 is restricted, he consumes his tax-adjusted endowment so that V_3 is affected only by τ_3. Therefore, the maximization problem can be re-written as

$$W(I) = \max_{\tau_3} \left\{ \left(\max_{\tau_1, \tau_2 | \tau_3} V_1 + V_2 \right) + V_3 \right\}$$

We need to show that for any given τ_3, we have $V_1 = V_2$ from the maximization problem, $\max_{\tau_1, \tau_2 | \tau_3} V_1 + V_2$. Given τ_3, the aggregate tax-adjusted endowments of Mr 1 and Mr 2 are fixed as $\omega_1 + \omega_2 + P(\alpha)\tau_3$ in state α and $\omega_1 + \omega_2 + P(\beta)\tau_3$ in state β. By the first welfare theorem applied in the tax-adjusted Edgeworth box, the trading equilibrium between Mr 1 and Mr 2 is Pareto optimal. Because they have identical homothetic vNM utility functions, the Pareto-optimal allocations satisfy the following:

$$\frac{x_1(\alpha)}{x_1(\beta)} = \frac{x_2(\alpha)}{x_2(\beta)} = \frac{\omega_1 + \omega_2 + P(\alpha)\tau_3}{\omega_1 + \omega_2 + P(\beta)\tau_3}$$

Define $t(\tau_3)$ as $(\omega_1 + \omega_2 + P(\alpha)\tau_3) / (\omega_1 + \omega_2 + P(\beta)\tau_3)$. Then, $V_1 + V_2$ can be expressed as

$$\begin{aligned} V_1 + V_2 &= \pi(\alpha) \log x_1(\alpha) + \pi(\beta) \log x_1(\beta) \\ &\quad + \pi(\alpha) \log x_2(\alpha) + \pi(\beta) \log x_2(\beta) \\ &= \pi(\alpha) \log x_1(\alpha) + \pi(\beta) \log t(\tau_3) x_1(\alpha) \\ &\quad + \pi(\alpha) \log x_2(\alpha) + \pi(\beta) \log t(\tau_3) x_2(\alpha) \\ &= \log x_1(\alpha) x_2(\alpha) + 2\pi(\beta) \log t(\tau_3). \end{aligned} \tag{4}$$

By the Second Welfare Theorem applied in the tax-adjusted Edgeworth box, any Pareto optimal allocation can be achieved by lump-sum transfers between Mr 1 and Mr 2. This implies that we have τ_1 and τ_2 such that $\tau_1 + \tau_2 = -\tau_3$

and that they maximize $\log x_1(\alpha) x_2(\alpha)$ in equation (4). Because $x_1(\alpha) + x_2(\alpha)$ is fixed at $\omega_1 + \omega_2 + P(\alpha)\tau_3$, the maximizing $x_1(\alpha)$ and $x_2(\alpha)$ are

$$x_1(\alpha) = x_2(\alpha) = \frac{\omega_1 + \omega_2 + P(\alpha)\tau_3}{2}. \tag{5}$$

Equation (5) also implies that $x_1(\beta) = x_2(\beta)$. Because $x_1(\alpha) = x_2(\alpha)$ and $x_1(\beta) = x_2(\beta)$, we have $V_1 = V_2$. ■

Lemma 2 $W(I) \geq W(R)$

Proof. From Lemma 2, welfare in the incomplete participation case can be expressed as

$$W(I) = \max_{\tau_3} \left\{ \left(\max_{\tau_1, \tau_2 | \tau_3} V_1 + V_2 \right) + V_3 \right\}. \tag{6}$$

On the other hand, in a fully-restricted market each consumer consumes his endowment directly. Therefore, V_h is a function of only τ_h. Then, welfare in the fully restricted case R can be expressed as

$$W(R) = \max_{\tau_1} V_1 + \max_{\tau_2} V_2 + \max_{\tau_3} V_3,$$

which is equivalent to

$$W(R) = \max_{\tau_3} \left\{ \left(\max_{\tau_1 | \tau_3} V_1 + \max_{\tau_2 | \tau_3} V_2 \right) + V_3 \right\}. \tag{7}$$

For any given τ_3, we know that

$$\left(\max_{\tau_1, \tau_2 | \tau_3} V_1 + V_2 \right) \geq \left(\max_{\tau_1 | \tau_3} V_1 + \max_{\tau_2 | \tau_3} V_2 \right). \tag{8}$$

From equations 6 and 7 and inequality 8, we have

$$\max_{\tau_3} \left\{ \left(\max_{\tau_1, \tau_2 | \tau_3} V_1 + V_2 \right) + V_3 \right\} \geq \max_{\tau_3} \left\{ \left(\max_{\tau_1 | \tau_3} V_1 + \max_{\tau_2 | \tau_3} V_2 \right) + V_3 \right\}.$$

■

5 Volatility and Social Welfare

In the case of perfect risk-sharing, price volatility does not affect social welfare. Welfare is at its maximum, independent of σ. However, in the cases where the securities market is not perfect and welfare is not at its maximum value, increased price volatility necessarily leads to decreased.

Proposition 3 *$W(I)$ and $W(R)$ are strictly decreasing in σ.*

Proof. Proposition 3 can be proved by Lemmas 3-4 ■

Lemma 3 *$W(I)$ is strictly decreasing in σ.*

Proof. Let $W^{\sigma}(I)$ be welfare at volatility σ. We need to establish that $W^{\sigma'}(I) > W^{\sigma''}(I)$ if $\sigma' < \sigma''$. $W^{\sigma}(I)$ can be expressed as (See Lemma 2)

$$W^{\sigma}(I) = \max_{\tau_3} \left\{ \left(\max_{\tau_1,\tau_2|\tau_3} V_1^{\sigma} + V_2^{\sigma} \right) + V_3^{\sigma} \right\},$$

where V_h^{σ} is Mr h's utility value with volatility σ. Define $T^{\sigma}(\tau_3)$ as

$$T^{\sigma}(\tau_3) = \left(\max_{\tau_1,\tau_2|\tau_3} V_1^{\sigma} + V_2^{\sigma} \right) + V_3^{\sigma}.$$

We need to show that for any value of τ_3, the following is true:

$$T^{\sigma'}(\tau_3) > T^{\sigma''}(\tau_3). \tag{9}$$

For given τ_3, we have $V_3^{\sigma'} > V_3^{\sigma''}$ where

$$V_3^{\sigma'} = \pi(\alpha)\log(\widetilde{\omega}_3(\alpha)) + \pi(\beta)\log(\widetilde{\omega}_3(\beta)),$$

because the log function is strictly concave and the tax-adjusted endowment with σ'' is mean-preserving spread of that with σ'.

We have

$$\begin{aligned} & \max_{\tau_1,\tau_2|\tau_3} V_1^{\sigma'} + V_2^{\sigma'} \qquad (10) \\ = \; & 2\left\{ \pi(\alpha)\log\left(\frac{\omega_1+\omega_2+P^m(\alpha)\tau_3}{2}\right) + \pi(\beta)\log\left(\frac{\omega_1+\omega_2+P^m(\beta)\tau_3}{2}\right)\right\} \end{aligned}$$

from Lemma 2. Since $P^m(s)$ is based on a mean-preserving spread, $\max_{\tau_1,\tau_2|\tau_3} V_1^{\sigma'} + V_2^{\sigma'}$ decreases because the log function in equation (10) is strictly concave Therefore, we have $T^{\sigma'}(\tau_3) > T^{\sigma''}(\tau_3)$ for all τ_3, which implies that

$$\max_{\tau_3} T^{\sigma'}(\tau_3) > \max_{\tau_3} T^{\sigma''}(\tau_3).$$

■

Lemma 4 $W(R)$ *is strictly decreasing in* σ.

Proof. For any given (τ_1, τ_2, τ_3), we know that each individual's expected utility function strictly decreases in σ because (1) vNM utility is strictly concave and (2) each individual's consumption is a mean-preserving spread increasing in σ. That is, for any balanced tax (τ_1, τ_2, τ_3) , we have

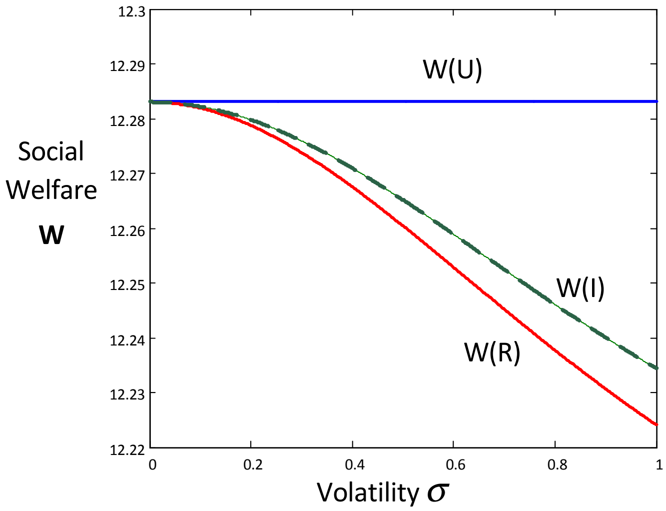
$$V_1^{\sigma'} + V_2^{\sigma'} + V_3^{\sigma'} > V_1^{\sigma''} + V_2^{\sigma''} + V_3^{\sigma''} \quad \text{if} \;\; \sigma' < \sigma''. \tag{11}$$

From equation (11), we have

$$\max_{\tau_1,\tau_2,\tau_3} V_1^{\sigma'} + V_2^{\sigma'} + V_3^{\sigma'} > \max_{\tau_1,\tau_2,\tau_3} V_1^{\sigma''} + V_2^{\sigma''} + V_3^{\sigma''}$$

if $\sigma' < \sigma''$. ■

In Figure 5, $W(U)$, $W(I)$ and $W(R)$ are plotted versus σ. $W(U)$ is invariant in σ but, while $W(I)$ and $W(R)$ are strictly decreasing in σ.



Simulation:
$(\omega_1, \omega_2, \omega_3) = (80, 60, 40), \pi(\alpha) = 0.5, P^m = 1$

6 Commodity Taxation

We assume that when the tax authority makes a net transfer of x units of chocolate from one consumer to another, δx units of the transferred chocolate are lost to "melting". The melting rate is $0 < \delta < 1$. If Mr h is taxed, i.e., $\tau_h^c > 0$, he owes the tax authority τ_h^c in chocolate. If consumer h is subsidized, i.e., $\tau_h^c < 0$, he will receive $(1 - \delta)\,\tau_h^c$ units of chocolate from the tax authority. Mr h's consumption is then

$$\omega_h - \max(0, \tau_h^c) - \min(0, (1 - \delta)\,\tau_h^c). \tag{12}$$

With the 3 consumers, maximized social welfare is

$$W = \max_{\tau_1^c, \tau_2^c, \tau_3^c} \sum_{h=1,2,3} E \log\left[\omega_h - \max(0, \tau_h^c) - \min(0, (1 - \delta)\,\tau_h^c)\right] \tag{13}$$

$$\text{subject to } \tau_1^c + \tau_2^c + \tau_3^c = 0.$$

First, we need to verify the conditions under which Mr 1 is taxed and Mr 3 is subsidized. Without loss of generality, we assume that $\omega_1 > \omega_2 > \omega_3$.

Lemma 5 *Mr 1 is taxed and Mr 3 is subsidized if and only if* $\delta < \frac{\omega_1 - \omega_3}{\omega_1}$*, i.e.,* $\omega_1 \geq \frac{\omega_3}{1-\delta}$.

Proof. Assume that Mr 1 is not taxed and Mr 3 is not subsidized. This implies that the tax authority cannot improve social welfare through a transfer from Mr 1 to Mr 3. The condition is this is

$$\left[\frac{\partial \log\left(\omega_1 - \tau_1^c\right)}{\partial \tau_1^c} + \frac{\partial \log\left(\omega_3 + (1-\delta)\,\tau_1^c\right)}{\partial \tau_1^c}\right]_{\tau_1^c=0} \leq 0,$$

which is equivalent to

$$\delta \geq 1 - \frac{\omega_3}{\omega_1} = \frac{\omega_1 - \omega_3}{\omega_1}.$$

Therefore, the condition that Mr 1 is taxed and Mr 3 is subsidized is

$$\delta < \frac{\omega_1 - \omega_3}{\omega_1}.$$

or,

$$\omega_1 \geq \frac{\omega_3}{1-\delta}.$$

■

Lemma 5 indicates that the tax authority taxes 1 chocolate from the rich to give $(1-\delta)$ chocolates to the poor until $\omega_3 = (1-\delta)\omega_1$. In the following Lemma, we verify the conditions under which Mr 2 is taxed or subsidized.

Lemma 6 *Mr 2 is subsidized if*

$$\omega_2 < \frac{1}{2}\left(\frac{\omega_1}{1-\delta} + \omega_3\right),$$

Mr 2 is taxed if

$$\omega_2 > \frac{1}{2}\left(\omega_1 + (1-\delta)\,\omega_3\right),$$

and Mr 2 is neither taxed nor subsidized if

$$\frac{1}{2}\left(\frac{\omega_1}{1-\delta} + \omega_3\right) \leq \omega_2 \leq \frac{1}{2}\left(\omega_1 + (1-\delta)\,\omega_3\right).$$

Proof. We assume that $\delta < (\omega_1 - \omega_3)\,\omega_1$. Then, Mr 1 is taxed and Mr 2 is subsidized by Lemma 5. Assuming that Mr 2 is neither taxed nor subsidized, we can derive the optimal tax θ_1^* for Mr. 1 from the following equation:

$$\frac{\partial \log(\omega_1 - \theta_1^*)}{\partial \theta_1} + \frac{\partial \log(\omega_3 + (1-\delta)\,\theta_1^*)}{\partial \theta_1} = 0,$$

which is equivalent to

$$\theta_1^* = \frac{1}{2}\frac{\omega_1 - \delta\omega_1 - \omega_3}{1-\delta} = \frac{1}{2}\left(\omega_1 - \frac{\omega_3}{1-\delta}\right).$$

Then, Mr 1's consumption x_1^* is

$$x_1^* = \omega_1 - \frac{1}{2}\left(\omega_1 - \frac{\omega_3}{1-\delta}\right) = \frac{1}{2}\omega_1 + \frac{1}{2}\frac{\omega_3}{1-\delta}.$$

Mr 3's consumption x_3^* is

$$x_3^* = \omega_3 + (1-\delta)\,\frac{1}{2}\left(\omega_1 - \frac{\omega_3}{1-\delta}\right) = \frac{1}{2}\left(\omega_1 + (1-\delta)\,\omega_3\right).$$

Because Mr 2 is not taxed, Mr 2's marginal cost of his commodity tax should be larger than the marginal benefit of Mr 3's additional subsidy:

$$\left[\frac{\partial \log(\omega_2 - \theta_2)}{\partial \theta_2} + \frac{\partial \log\left(\frac{1}{2}\left(\omega_1 + (1-\delta)\,\omega_3\right) + (1-\delta)\,\theta_2\right)}{\partial \theta_2}\right]_{\theta_2 = 0} < 0,$$

which is equivalent to

$$\omega_2 < \frac{\frac{1}{2}\left(\omega_1 + (1-\delta)\,\omega_3\right)}{(1-\delta)} = \frac{1}{2}\left(\frac{\omega_1}{1-\delta} + \omega_3\right) \tag{14}$$

This is the condition for when Mr 2 is not taxed. The condition where Mr 2 is not subsidized can be derived in the same way. Thus we have

$$\omega_2 > \frac{1}{2}\left(\omega_1 + (1-\delta)\,\omega_3\right). \tag{15}$$

From conditions (14) and (15), we can derive the condition for when Mr 2 is neither taxed nor subsidized. That is,

$$\frac{1}{2}\left(\omega_1 + (1-\delta)\,\omega_3\right) < \omega_2 < \frac{1}{2}\left(\frac{\omega_1}{1-\delta} + \omega_3\right) \tag{16}$$

■

We can interpret the tax authority's problem geometrically: Maximize $x_1 x_2 x_3$ subject to a "budget set" with a kink at the endowment point.

7 Social Welfare and Commodity Taxation

Proposition 4 *Social welfare is strictly decreasing in δ if $(\tau_1^c, \tau_2^c, \tau_3^c) \neq 0$.*

Proof. Define $T(\tau^c, \delta)$ by

$$T(\tau^c, \delta) = \sum_{h=1,2,3} E \log\left[\omega_h - \max(0, \tau_h^c) - \min(0, (1-\delta)\,\tau_h^c)\right]. \tag{17}$$

Then, maximized social welfare is

$$W = \max_{\tau_1^c, \tau_2^c, \tau_3^c} T(\tau^c, \delta).$$

For any $(\tau_1^c, \tau_2^c, \tau_3^c)$, we have

$$T(\tau^c, \delta') > T(\tau^c, \delta'') \; if \; \delta' < \delta''$$

because $T(\tau^c, \delta)$ is decreasing in δ if $\tau_h^c < 0$ for some h in equation (17). Therefore, we have

$$\max_{\tau_1^c, \tau_2^c, \tau_3^c} T(\tau^c, \delta') > \max_{\tau_1^c, \tau_2^c, \tau_3^c} T(\tau^c, \delta'') \quad \text{if} \quad \delta' < \delta''.$$

■

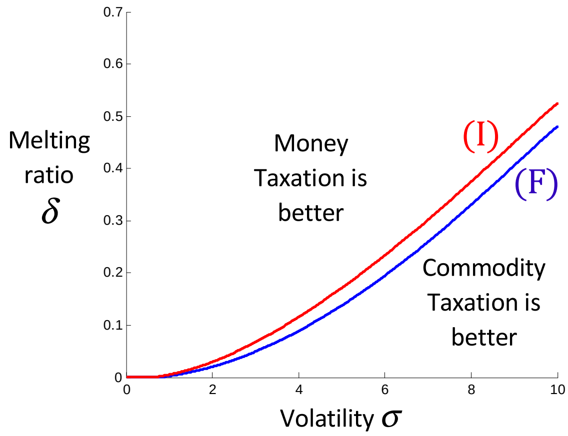
8 Money taxation vs. Commodity taxation

From Lemmas 5 and 4, commodity taxation welfare reaches its first-best value $3\log\left(\frac{\omega_1+\omega_2+\omega_3}{3}\right)$ when $\delta = 0$, strictly decreases in δ, and reaches it minimum value of $\log \omega_1\omega_2\omega_3$ when $\delta = \frac{\omega_1-\omega_3}{\omega_1}$. On the other hand, with money taxation, social welfare never falls below its minimum because even with high volatility σ, the marginal cost of taxation of the rich never exceeds the marginal benefit of subsidizing for the poor in autarky (i.e. when $\tau = 0$). Therefore, we have the following proposition:

Proposition 5 *If the tax authority can choose either money taxation and commodity taxation, for any given volatility level σ, there exists $\delta^* \in \left(0, \frac{\omega_1-\omega_3}{\omega_1}\right)$ such that welfare with money taxation (under partially or fully restricted markets) is higher (lower) than welfare with commodity taxation if $\delta > (<)\delta^*$. δ^* is strictly decreasing in σ. For any given volatility level σ, the value of δ^* under the partially restricted market is higher than that under the fully restricted market. δ^* is a different fraction depending on whether the money taxation economy is partially restricted or fully restricted.*

Proof. Directly from Propositions 1, 3 and 4 and Lemma 5. ■

In the plot in Figure 8, (δ, σ)-space is divided into a region in which dollar taxation is better and another region in which chocolate taxation is better. The region in which dollar taxation is better for partially restricted market participation is a subset of the corresponding set for fully restricted market participation.



Simulation: $(\omega_1, \omega_2, \omega_3) = (100, 50, 10), \pi(\alpha) = \pi(\beta) = 0.5, P^M = 10$

9 Concluding remarks

We weigh the advantages and disadvantages of a simple finance economy (the money-taxation regime) against those of the corresponding non-finance economy (the commodity-taxation regime). Taxes are endogenous. They are chosen optimally by the tax authority. The desirability of the money-taxation regime is declining in the volatility of the price level. The desirability of the commodity-taxation regime is declining in the iceberg-style costs of net tax transfers. In the money-taxation regime, the tax authority equalizes the expected utilities of all those with access to the security market. In the commodity-taxation regime, the tax authority equalizes the utilities of the taxed consumers and equalizes the utilities of the subsidized consumers.

The model allows for information frictions in which some or all of the consumers are restricted from participation on the securities market. When these restrictions are absent, the money-tax economy achieves the first-best allocation, in which all utilities are equalized. Otherwise, social welfare is strictly decreasing in price-level volatility.

The effects of volatility on individual expected utilities are more complicated and worthy of separate study. There are several effects (1) the direct effects on tax adjusted endowments, which in general become more volatile as price-level volatility increases, (2) the hedging effects through the securities market, (3) the effects of volatility on the tax authority's choice of tax regime and its choice of taxes. The third effect would not be present if — as in the existing literature — taxes were predetermined independently of volatility. Some individuals are harmed by volatility, but others might be made better off from volatility for at least two reasons: (1) Taxed individuals might benefit as the tax authority reduces taxation because of increased social costliness as increased volatility causes tax-adjusted endowments to become more volatile; (2) Some consumers might benefit from volatility by sharing through the market the increased risks of other consumers.

References

Aumann, R. J. (1987). Correlated equilibrium as an expression of bayesian rationality. *Econometrica 55*(1), 1–18.

Aumann, R. J., J. Peck, and K. Shell (1985, Revised 1988). *Asymmetric information and sunspot equilibria: a family of simple examples.* Working Paper 88-34, Center for Analytic Economics, Cornell University.

Balasko, Y. and K. Shell (1993). Lump-sum taxation: the static economy. In R. Becker, M. Boldrin, R. Jones, and W. Thomson (Eds.), *General equilibrium, growth and trade. Essays in honor of Lionel McKenzie*, Volume 2, pp. 168–180. San Diego, CA: Academic Press.

Bhattacharya, J., M. G. Guzman, and K. Shell (1998). Price level volatility: A simple model of money taxes and sunspots. *Journal of Economic Theory 81*(2), 401–430.

Cass, D. and K. Shell (1983). Do sunspots matter? *Journal of Political Economy 91*(2), 193–227.

Cozzi, G., A. Goenka, M. Kang, and K. Shell (2015). Winners and losers from price-levelvolatility: Money taxation and information frictions. *CAE Working Paper #15-01*, Cornell University.

Goenka, A. (1994). Rationing and sunspot equilibria. *Journal of Economic Theory 64*(2), 424– 442.

Peck, J. and K. Shell (1991). Market uncertainty: correlated and sunspot equilibria in imperfectly competitive economies. *Review of Economic Studies 58*(5), 1011–1029.