three periods: $T = 0, 1, 2$

- a single good

- a continuum of agents with measure 1

- Each agent is endowed with 1 unit of the good in period 0.
The Model: Asset Return

\[ T = 0 \quad T = 1 \quad T = 2 \]

\[
-1 \quad \begin{cases} 
0 \\
R \\
0 
\end{cases} 
\]
In period 0, all agents are identical.

In period 1, some agents become “patient” and others become “impatient”. (private information)

\[
\begin{cases}
  u(c_1) & \text{if impatient} \\
  u(c_2) & \text{if patient}
\end{cases}
\]

The probability of being impatient is \( \lambda \) for each agent in period 0.
Autarky

- autarky:
  - utility of the impatient in period 1: $u(1)$
  - utility of the patient in period 2: $u(R)$
  - expected utility in period 0: $\lambda u(1) + (1 - \lambda) u(R)$

- $1 < R$
  - “insurance” against the liquidity shock is desirable.
Banking Economy

- Banks offers demand deposit contract \((d_1, d_2)\).
- Agents
  - make deposits in period 0.
  - withdraw \(d_1\) in period 1.
  - or withdraw \(d_2\) in period 2.
- Free-entry banking sector: \((d_1, d_2)\) maximizes the depositor’s expected utility.
Optimal Deposit Contract

\[
\max_{d_1, d_2} \lambda u(d_1) + (1 - \lambda) u(d_2)
\]

s.t.
\[
(1 - \lambda)d_2 \leq (1 - \lambda d_1)R \quad (RC)
\]

withdrawals in period 2 \quad resources in period 2

\[
d_1 \leq d_2 \quad (IC)
\]
Optimal Deposit Contract:

\[(1 - \lambda)d_2 = (1 - \lambda d_1)R\]

slope = \[-\frac{\lambda}{1 - \lambda}R\]

45°
Optimal Deposit Contract:

\[ \lambda u(d_1) + (1 - \lambda)u(d_2) = \text{const} \]

slope = \(-\frac{\lambda}{1 - \lambda} \frac{u'(d_1^*)}{u'(d_2^*)}\)

\[ (1 - \lambda)d_2 = (1 - \lambda d_1)R \]

slope = \(-\frac{\lambda}{1 - \lambda} R\)

\[ \frac{\lambda}{1 - \lambda} \frac{u'(d_1^*)}{u'(d_2^*)} = \frac{\lambda}{1 - \lambda} R \]

\[ \text{MRS} = \frac{\lambda}{1 - \lambda} \frac{R}{\text{MRT}} \]
What do banks do?

- $u'(d_1^*) / u'(d_2^*) = R$
- $u'' < 0 \Rightarrow d_1^* < d_2^*$
- CRRA: $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$
  - $u'(c) = c^{-\gamma} \Rightarrow u'(d_1) / u'(d_2) = (d_2 / d_1)^\gamma$
  - if $\gamma = 1 \Rightarrow d_1^* = 1, d_2^* = R$
  - if $\gamma > 1 \Rightarrow 1 < d_1^* < d_2^* < R$
Why do bank runs occur?

- $\gamma > 1 \implies 1 < d_1^* < d_2^* < R$
- IC: $d_1 \leq d_2$
- Expectation: Only the impatient depositors withdraw in period 1.
- A patient depositor can
  \[
  \begin{cases}
  \text{get } d_2^* & \text{if he withdraws in period 2} \\
  \text{get } d_1^* & \text{if he withdraws in period 1}
  \end{cases}
  \]
Why do bank runs occur?

- $\gamma > 1 \iff 1 < d_1^* < d_2^* < R$
- Expectation: All other depositors demand withdraw in period 1.
- A patient depositor can
  \[
  \begin{cases}
  \text{get nothing} & \text{if he withdraws in period 2} \\
  \text{get } d_1^* \text{ w.p. } (1/d_1^*) & \text{if he withdraws in period 1}
  \end{cases}
  \]