Connections between Futures Market Economy and Money Market Economy

One good per period, $\ell = 1$, two periods, $t = 1, 2$.

**Futures Market:**

$$\max_u \left( x_h^1, x_h^2 \right)$$

$s.t.$ $p^1 x_h^1 + p^2 x_h^2 = p^1 \omega_h^1 + p^2 \omega_h^2$

Equilibrium is a price vector $(p^1, p^2)$ such that

$$\sum_h x_h^t = \sum_h \omega_h^t \text{ for } t = 1, 2.$$

Define the interest factor $R$ and the interest rate $r$ in terms of the equilibrium commodity prices $(p^1, p^2)$.

**Money Market:**

$$\max_u \left( x_h^1, x_h^2 \right)$$

$s.t.$ $p^1 x_h^1 + p^m^1 m_h^1 = p^1 \omega_h^1$

$$p^2 x_h^2 + p^m^2 m_h^2 = p^2 \omega_h^2$$

Equilibrium $(p^1, p^2, p^m^1, p^m^2)$ such that

$$\sum_h x_h^t = \sum_h \omega_h^t \text{ and } \sum_h m_h^t = 0 \text{ for } t = 1, 2.$$
1.

Prove that in equilibrium \( p^{m_1} = p^{m_2} = p^m \geq 0 \). This is a no-arbitrage-property result.

2.

Show that if, \( x_{h_1}, x_{h_2} \), \( h = 1, ..., n \) solves the futures market problem, it also solves the money market problem.

3.

Show that if, \( (x_{h_1}, x_{h_2}) \), \( h = 1, ..., n \) solves the money market problem with \( p^m > 0 \), then it also solves the futures market problem.