Economics 4905
Financial Fragility & the Macroeconomy
Fall 2015
Problem Set #2
Due Monday, October 6, 2015

Connections between Futures Market Economy and Money Market Economy

One good per period, $\ell = 1$, two periods, $t = 1, 2$.

Futures Market:

$$\max \ \mathrm{u}_h\left(x_h^1, x_h^2\right)$$
$$s.t.\ p^1 x_h^1 + p^2 x_h^2 = p^1 \omega_h^1 + p^2 \omega_h^2$$

Equilibrium is a price vector $\left(p^1, p^2\right)$ such that

$$\sum_h x_h^t = \sum_h \omega_h^t \text{ for } t = 1,\ 2.$$

Define the interest factor R and the interest rate r in terms of the equilibrium commodity prices $\left(p^1, p^2\right)$.

Money Market:

$$\max \ \mathrm{u}_h\left(x_h^1, x_h^2\right)$$
$$s.t. \quad \begin{aligned} p^1 x_h^1 + p^{m^1} m_h^1 &= p^1 \omega_h^1 \\ p^2 x_h^2 + p^{m^2} m_h^2 &= p^2 \omega_h^2 \end{aligned}$$

Equilibrium $\left(p^1, p^2, p^{m^1}, p^{m^2}\right)$ such that

$$\sum_h x_h^t = \sum_h \omega_h^t \text{ and } \sum_h m_h^t = 0 \text{ for } t = 1,\ 2.$$

1.

Prove that in equilibrium $p^{m^1} = p^{m^2} = p^m \geq 0$. This is a no-arbitrage-property result.

2.

Show that if, $\left(x_h^1, x_h^2\right)$, $h = 1, \ldots, n$ solves the futures market problem, it also solves the money market problem.

3.

Show that if, $\left(x_h^1, x_h^2\right)$, $h = 1, \ldots, n$ solves the money market problem with $p^m > 0$, then it also solves the futures market problem.