How Optimal Banking Contracts Tolerate Runs

Karl Shell       Yu Zhang

Cornell University    Xiamen University

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Introduction

- Bryant (1980) and Diamond and Dybvig (1983): “bank runs” in the *post-deposit* game
- Peck and Shell (2003): A *sunspot-driven* run can be an equilibrium in the *pre-deposit* game for sufficiently small run probability.
For the 2-depositor banking model, the optimal contract is defined by $c$ – the withdrawal of the first in line in period 1.
Introduction

- Shouldn’t $c^*(s)$ become more conservative (i.e., strictly decreasing) in $s$ before it switches to the best run-proof contract?
- If yes, in which economies will we have this property and in which economies is $c^*(s)$ a step function?
- These issues are important to banks and regulators. Also important to the theory of SSE.
- Instead of relying solely on numerical examples, we provide the global comparative statics for this economy.
The Model: Consumers

- 2 ex-ante identical vNM consumers and 3 periods: 0, 1 and 2.
- Endowments: \( y \)
- Preferences: \( u(c^1) \) and \( v(c^1 + c^2) \)

\[
u(x) = A \frac{(x)^{1-a}}{1-a}, \text{ where } A > 0 \text{ and } a > 1.
\]

\[
v(x) = \frac{(x)^{1-b}}{1-b}, \text{ where } b > 1.
\]

- Types are uncorrelated (so we have aggregate uncertainty.): \( p \)
The Model: Technology

- Bank Portfolio:

\[
\begin{array}{ccc}
  t = 0 & t = 1 & t = 2 \\
  -1 & 1 & 0 \\
  -1 & 0 & R
\end{array}
\]

- Consumer storage option
The Model

- Sequential service constraint (Wallace (1988))
- Suspension of convertibility.
- A depositor visits the bank only when he makes withdrawals.
- When a depositor makes his withdrawal decision, he does not know his position in the bank queue.
- If more than one depositor chooses to withdraw, a depositor’s position in the queue is random. Positions in the queue are equally probable.
Post-Deposit Game: Notation

- $c \in [0, 2y]$ is any feasible banking contract
- $\widehat{c} \in [0, 2y]$ is the unconstrained optimal banking contract
- $c^* \in [0, 2y]$ is the constrained optimal banking contract
early

- A patient depositor chooses early withdrawal when he expects the other depositor, if patient, to also choose early withdrawal.

\[
\frac{v(c) + v(2y - c)}{2} > v[(2y - c)R]
\]

- Let \( c^{early} \) be the value of \( c \) such that the above inequality holds as an equality. \( c^{early} \) is the best run-proof \( c \).
A patient depositor chooses late withdrawal when he expects the other depositor, if patient, to also choose late withdrawal. (ICC)

\[ pv[(2y - c)R] + (1 - p)v(yR) \geq p[v(c) + v(2y - c)]/2 + (1 - p)v(c). \]

Let \( c^{wait} \) be the value of \( c \) such that the above inequality holds as an equality.
The post-deposit game has two equilibria: one run and one non-run.

0 \quad c^{\text{early}} \quad c^{\text{wait}} \quad 2y

Only the non-run equilibrium exists. Only the run equilibrium exists.
Post-Deposit Game

- $c^{early} < c^{wait}$ if and only if

$$b < \min\{2, 1 + \ln 2 / \ln R\}$$

- When $b$ and $R$ satisfy the above inequality, bank runs matter in the *post-deposit* game for $c \in (c^{early}, c^{wait}]$.

- When $b$ and $R$ don’t satisfy the above inequality, $c^{early} \geq c^{wait}$, which implies that any implementable allocation is strongly implementable; hence bank runs do not matter.
Whether bank runs occur in the pre-deposit game depends on whether the optimal contract $c^*$ belongs to the set $(c^{early}, c^{wait}]$.

To characterize the optimal contract, we divide the problem into three cases depending on $\hat{c}$, the contract supporting the unconstrained efficient allocation.

- $\hat{c} \leq c^{early}$ (Case 1)
- $\hat{c} \in (c^{early}, c^{wait}]$ (Case 2)
- $\hat{c} > c^{wait}$ (Case 3)
Impulse parameter A and the 3 cases

- $\hat{c}$ is the $c$ in $[0, 2y]$ that maximizes

$$\hat{W}(c) = p^2[u(c) + u(2y - c)] + 2p(1 - p)[u(c) + v((2y - c)R)] + 2(1 - p)^2v(yR).$$

- $\hat{c} = \frac{2y}{\left\{p/(2 - p) + 2(1 - p)/[(2 - p)AR^{b-1}]\right\}^{1/b} + 1}.$

- $\hat{c}(A)$ is an increasing function of $A.$
Parameter A and the 3 Cases

- Neither $c^{\text{early}}$ nor $c^{\text{wait}}$ depends on $A$

Case 2: Unconstrained efficient allocation is not uniquely implementable.

Case 1: Unconstrained efficient allocation is uniquely implementable.

Case 3: Unconstrained efficient allocation is not implementable.
Example

- The parameters are

\[ a = b = 1.01; \ p = 0.5; \ y = 3; \ R = 1.5 \]

- We see that \( b \) and \( R \) satisfy the condition which makes the set of contracts permitting strategic complementarity non-empty. We have that \( c^{\text{early}} = 4.155955 \) and \( c^{\text{wait}} = 4.280878 \).

- \( A_1 = 6.217686 \) and \( A_2 = 10.277988 \).

- If \( A \leq A_1 \), we are in Case 1; If \( A_1 < A \leq A_2 \), we are in Case 2; If \( A > A_2 \), we are in Case 3.
The Optimal Contract

\[ c^*(s) = \arg \max_{c \in [0, c^{\text{wait}}]} W(c; s), \]

where

\[ W(c; s) = \begin{cases} 
\hat{W}(c) & \text{if } c \leq c^{\text{early}}. \\
(1 - s)\hat{W}(c) + sW^{\text{run}}(c) & \text{if } c^{\text{early}} < c \leq c^{\text{wait}}. 
\end{cases} \]

and

\[ W^{\text{run}}(c) = p^2 [u(c) + u(2y - c)] \\
+ p(1 - p)[u(c) + \nu(2y - c) + \nu(c) + u(2y - c)] \\
+ (1 - p)^2 [\nu(c) + \nu(2y - c)]. \]
The Optimal Contract

The post-deposit game has two equilibria: one run and one non-run.

Only the non-run equilibrium exists.

Only the run equilibrium exists.
The Optimal Contract: Case 1

- Case 1: The *unconstrained efficient allocation* is strongly implementable, i.e., $\hat{c} \leq c^{\text{early}}$.

- It is straightforward to see that the optimal contract for the *pre-deposit* game supports the *unconstrained efficient allocation* $c^*(s) = \hat{c}$.

and that the optimal contract doesn’t tolerate runs.
Case 2: The *unconstrained efficient allocation* is weakly implementable, i.e., \( c^{early} < \hat{c} \leq c^{wait} \).

The optimal contract \( c^*(s) \) satisfies: (1) if \( s \) is larger than the threshold probability \( s_0 \), the optimal contract is run-proof and \( c^*(s) = c^{early} \). (2) if \( s \) is smaller than \( s_0 \), the optimal contract \( c^*(s) \) tolerates runs and it is a strictly decreasing function of \( s \).
The Optimal Contract: Case 2

- Using the same parameters as the previous example. Let \( A = 8 \). (We have seen that we are in Case 2 if \( 6.217686 < A \leq 10.277988 \).
- \( c^* \) switches to the best run-proof contract (i.e. \( c^{\text{early}} \)) when \( s > s_0 = 0.001382358 \).
Case 3: The unconstrained efficient allocation is not implementable, i.e., $c^{\text{wait}} < \hat{c}$.

The optimal contract $c^*(s)$ satisfies: (1) If $s$ is larger than the threshold probability $s_1$, we have $c^*(s) = c^{\text{early}}$ and the optimal contract is run-proof. (2) If $s$ is smaller than $s_1$, the optimal contract $c^*(s)$ tolerates runs and it is a weakly decreasing function of $s$. Furthermore, we have $c^*(s) = c^{\text{wait}}$ for at least part of the run tolerating range of $s$. 
The Optimal Contract: Case 3

- Using the same parameters as in the previous example. Let $A = 10.4$. (We have seen that we are in Case 2 if $A > 10.277988$.)
- $c^*$ switches to the best run-proof (i.e. $c^{early}$) when $s > 0.004524181$.
- ICC becomes non-binding when $s \geq 0.001719643$.
The Optimal Contract: Case 3

- Let $A = 11$. (PS case)
- $c^*$ switches to the best run-proof (i.e. $c^{early}$) when $s > 0.005281242$. 

![Figure 5. $c^*(s)$ for $A=11$](image-url)
The Optimal Contract

- \( c^* \) versus \( s \) and \( A \)
The Optimal Contract

- welfare loss from using the corresponding optimal bang-bang contract instead of $c^*(s)$
The general form of the optimal contract to the \textit{pre-deposit} game is analyzed.

The \textit{unconstrained efficient allocation} falls into one of the three cases:

- (1) strongly implementable
- (2) weakly implementable
- (3) not implementable.
Summary and Concluding Remark

- In Cases 2 and 3, the optimal contract tolerates runs when the run probability is sufficiently small:

- In Case 2, the optimal contract adjusts continuously and becomes strictly more conservative as the run probabilities increases.

  - The optimal allocation is never a mere randomization over the *unconstrained efficient allocation* and the corresponding run allocation from the *post-deposit* game. Hence this is also a contribution to the sunspots literature: another case in which SSE allocations are not mere randomizations over certainty allocations.
In Case 3, the ICC binds for small run-probabilities, which forces the contract to be more conservative than it would have been without the ICC. Hence, for Case 3, the optimal contract does not change with $s$ until the ICC no longer binds.

For small $s$, the optimal allocation is a randomization over the constrained efficient allocation and the corresponding run allocation from the post-deposit game.