Economics 4905
Financial Fragility & the Macroeconomy
Fall 2015
Solutions to Problem Set #2
Due Monday, October 6, 2015
Revised October 25, 2015

<u>**Connections between Futures Market Economy and Money Market Economy**</u>

One good per period, $\ell = 1$, two periods, $t = 1, 2$.

<u>Futures Market:</u>

$$\max \ u_h\left(x_h^1, x_h^2\right)$$
$$s.t.\ p^1 x_h^1 + p^2 x_h^2 = p^1 \omega_h^1 + p^2 \omega_h^2$$

Equilibrium is a price vector $\left(p^1, p^2\right)$ such that

$$\sum_h x_h^t = \sum_h \omega_h^t \text{ for } t = 1,\ 2.$$

Define the interest factor R and the interest rate r in terms of the equilibrium commodity prices $\left(p^1, p^2\right)$.

Money Market:

$$\max\ \mathrm{u}_h\left(x_h^1, x_h^2\right)$$

$$s.t.\quad \begin{array}{c} p^1 x_h^1 + p^{m^1} m_h^1 = p^1 \omega_h^1 \\ p^2 x_h^2 + p^{m^2} m_h^2 = p^2 \omega_h^2 \end{array}$$

Equilibrium $\left(p^1, p^2, p^{m^1}, p^{m^2}\right)$ such that

$$\sum_h x_h^t = \sum_h \omega_h^t \text{ and } \sum_h m_h^t = 0 \text{ for } t = 1,\ 2.$$

0.

From the definitions of present prices, the interest factor and the interest rate, we have:

$$p^1 = 1,\ \left(p^1,\ \mathrm{p}^2\right) = \left(1,\ \mathrm{p}^2\right).$$

$$p_2 = \frac{1}{R} = \frac{1}{1+r}.$$

1.

Prove that in equilibrium $p^{m^1} = p^{m^2} = p^m \geq 0$. This is a no-arbitrage-property result.

Answer:

Consumer h maximizes his financial wealth through arbitrage:

$$\max W_h = p^{m^1} m_h^1 + p^{m^2} m_h^2$$

$$s.t. \quad \begin{aligned} m_h^1 + m_h^2 &= 0 \\ m_h^2 &= -m_h^1. \end{aligned}$$

$$W_h = \left(p^{m^1} - p^{m^2}\right) m_h^1.$$

If $p^{m^1} > p^{m^2}$, $m_h^1 > 0$.
Borrow (high) in period 1 and re-pay (low) in period 2.

If $p^{m^2} > p^{m^1}$, $m_h^1 > 0$.

If $p^{m^2} \neq p^{m^1}$, W_h can be arbitrarily large by choosing $\left|m_h^1\right|$ arbitrarily large. This allows x_h^1 and x_h^2 to be arbitrarily large.

So,

$$\sum_h x_h^t > \sum_h \omega_h^t \text{ for } t = 1, 2.$$

Hence, $p^{m^2} = p^{m^1}$ is required for competitive equilibrium. Let

$$p^m = p^{m^1} = p^{m^2} \geq 0.$$

2.

Show that if, $\left(x_h^1, x_h^2\right)$, $h = 1, \ldots, n$ solves the futures market problem, it also solves the money market problem.

Proof:

Excess demand is $z_h^t = x_h^t - \omega_h^t$. $p^1 = 1$.

$$z_h^1 + p^2 z_h^2 = 0$$
$$z_h^1 = -p^2 z_h^2$$

Define the scalar k by:

$$k = z_h^1 - p^2 z_h^2 .$$

Set

$$k = p^{m^1} m_h^1 = -p^m m_h^2 .$$

Hence,

$$z_h^1 + p^m m_h^2 = 0$$
$$p^2 z_h^2 + p^m m_h^2 = 0$$
$$m_h^1 + m_h^2 = 0.$$

Hence if $\left(x_h^1, x_h^2\right)$ is an equilibrium allocation in the futures market, is is also an equilibrium in the money market.

3.

Show that if, $\left(x_h^1, x_h^2\right)$, $h = 1, \ldots, n$ solves the money market problem with $p^m > 0$, then it also solves the futures market problem.

$$m_h^1 + m_h^2 = 0,\ m_h^2 = -m_h^1.$$

$$p^m > 0.$$

$$z_h^1 = -p^m m_h^1 \ , \quad p^2 z_h^2 = p^m m_h^1.$$

$$\frac{z_h^1}{p^m} = -m_h^1 \ , \qquad \frac{p^2 z_h^2}{p^m} = m_h^1.$$

m_h^1 is a slack variable.

Hence, we have

$$z_h^1 + p^2 z_h^2 = 0.$$

Hence, if $\left(x_h^1, x_h^2\right)$ is an equilibrium allocation in the money market economy with , then $\left(x_h^1, x_h^2\right)$ is also an equilibrium allocation in the corresponding futures market economy.