Economics 6130
Cornell University
Fall 2015
Macroeconomics I - Part 2

Solutions to Problem Set #2

Question 1. Static one-good ($l = 1$) pure-exchange economy with money taxes and transfers. Four consumers ($n = 4$) .

$$\omega = (\omega_1, \omega_2, \omega_3, \omega_4) = (10, 8, 4, 2)$$

where $\omega_i > 0$ is the endowment of consumer i ($i = 1, 2, 3, 4$).
For each of the following cases: Is $\tau = (\tau_1, \tau_2, \tau_3, \tau_4)$ balanced? Is τ bonafide? Describe the set of equilibrium money prices.

(a) $\tau = (2, 1, -1, -1)$

(b) $\tau = (5, 2, -3, -4)$

(c) $\tau = (4, -1, 1, -2)$

(d) $\tau = (5, -4, 1, -2)$

(e) $\tau = (0, 0, 0, 0)$

Solution:

(a) No, $\sum_h \tau_h = 1 > 0$. Therefore taxes aren't balanced, and therefore aren't bonafide. In equilibrium we must have $P^m \sum_h \tau_h = 0$, thus we must have $P^m = \{0\}$ as the equilibrium set of money prices.

(b) Yes $\sum_h \tau_h = 0$, so τ is balanced, and therefore bonafide. We know that the equilibrium set of money prices is decided by $P^m < \min_h \left[\frac{\omega_h}{\max(0, \tau_h)}\right]$. So we have $P^m \in [0, 2)$.

(c) No, $\sum_h \tau_h = 2 > 0$. Same as (1a).

(d) Yes, τ is balanced and therefore bonafide. And we get the equilibrium set of money prices to be $P^m \in [0, 2)$.

(e) Yes, τ is balanced and bonafide, but note that for all h we have $\max(0, \tau_h) = 0$, therefore we can have any equilibrium money price. So $P^m \in [0, \infty)$.

Question 2. Static one-good ($l = 1$) pure-exchange economy. Three consumers ($n = 3$). Endowments are given by

$$\omega = (\omega_1, \omega_2, \omega_3) = (3, 8, 3)$$

Two currencies, R (for Red) and B (for Blue). Let $p^{mR} \geq 0$ and $p^{mB} \geq 0$ be the goods-price of red and blue money, respectively.

For each case (a) through (d), calculate the set of equilibrium prices of red money, blue money, and the equilibrium exchange rate between the two currencies. Do not ignore cases in which one or both moneys are worthless. Please be very careful of units.

(a) $\tau_R = (2, 0, 2),\ \tau_B = (-3, 3, -3)$

(b) $\tau_R = (-1, -1, 2),\ \tau_B = (2, -1, -1)$

(c) $\tau_R = (-2, 7, -2),\ \tau_B = (-1, 0, -1)$

(d) $\tau_R = (2, -2, 2),\ \tau_B = (-1, 4, -2)$

What are the economics behind your answers? Interpret your answers as if R = dollars and B = euros. How are the budget deficits and trade deficits related in this model?

What do you make of the fact that money prices and exchange rates are not always uniquely determined?

Solution:

(a) Solving (1) yields $p^{mR} = \frac{3}{4}p^{mB}$, so $XR_{RB} = \frac{3}{4}\frac{B\$}{R\$}$. Therefore neither currency can be worthless, unless both are worthless. For each Mr. h we must have that $x_h = \omega_h - p^{mR}\tau_h^R - p^{mB}\tau_h^B > 0$. For Mr. 2 this implies $8 - 3p^{mB} > 0 \Leftrightarrow \frac{8}{3} > p^{mB}$. So ensuring that each consumer has positive consumption implies that $p^{mB} \in [0, 8/3)$. From $p^{mR} = \frac{3}{4}p^{mB}$ it follows that $p^{mB} \in [0, 2)$.

(b) Both tax systems are balanced, therefore we know that no matter the prices of each type of money we will always have

$$p^{mR} \sum_h \tau_h^R + p^{mB} \sum_h \tau_h^B = 0 \tag{1}$$

If the other money is worthless, then by the approach from Question 1, the equilibrium prices to be either $p^{mR} \in [0, \frac{3}{2})$ or $p^{mB} \in [0, \frac{3}{2})$. Now suppose that neither money is worthless. Then Mr. 2 is always subsidized and hence can't be pushed out of his consumption set. For Mr. 1 and Mr. 3 we must have that $x_h = \omega_h - p^{mR}\tau_h^R - p^{mB}\tau_h^B > 0$. This implies we must satisfy the following

$$3 > -p^{mR} + 2p^{mB}$$
$$3 > 2p^{mR} - p^{mB}$$

Combining and solving we get that the equilibrium money prices must satisfy that $p^{mR} < \frac{3}{2}$ and $p^{mB} \in (2p^{mR} - 3, \frac{3+p^{mR}}{2})$.

But this doesn't exclude any exchange rates, hence the equilibrium exchange rates can be anything such that $XR_{RB} \in [0, \infty)$.

(c) Solving as in part (a) $p^{mR} = \frac{2}{3}p^{mB}$, so $XR_{RB} = \frac{2}{3}\frac{B\$}{R\$}$. Mr 1. and Mr. 3 are always subsidized, and keeping Mr. 2's consumption set positive requires that the equilibrium money prices satisfy $p^{mR} \in [0, \frac{8}{7})$. It follows from $p^{mR} = \frac{2}{3}p^{mB}$ that $p^{mB} \in [0, \frac{12}{7})$.

(d) Since both tax policies run a surplus we get that we must in equilibrium have that $p^{mR} = p^{mB} = 0$. And we can't determine an equilibrium exchange rate.

To get an equilibrium exchange rate, we must always have one currency running a surplus and the other a deficit. Otherwise money will either be worthless, or if they are balanced there will be many exchange rates possible.

Question 3. Offer curve. Single consumer. Two commodities.

$$u(x_1, x_2) = \log x_1 + \beta \log x_2$$
$$p = \frac{p_2}{p_1}$$
$$\omega_1 = 150,\ \omega_2 = 75$$
$$\beta = 0.98$$
$$z_1 = x_1 - \omega_1,\ z_2 = x_2 - \omega_2$$

Calculate and graph the offer curve (OC) in (z_1, z_2) space.

Solution: Our problem is

$$\max_{x_1, x_2} \log x_1 + \beta \log x_2$$
$$\text{s.t. } x_1 + px_2 \leq \omega_1 + p\omega_2$$

Taking first order conditions and dividing them yields $x_1 = \frac{p}{\beta} x_2$. Plugging into the budget constraint yields the demand for each in terms of p:

$$x_1 = \frac{\omega_1 + p\omega_2}{1 + \beta}$$
$$x_2 = \frac{\beta(\omega_1 + p\omega_2)}{p(1 + \beta)}$$

Expressed in terms of excess demand:

$$z_1 = \frac{p\omega_2 - \beta\omega_1}{1 + \beta}$$
$$z_2 = \frac{\beta\omega_1 - p\omega_2}{p(1 + \beta)}$$

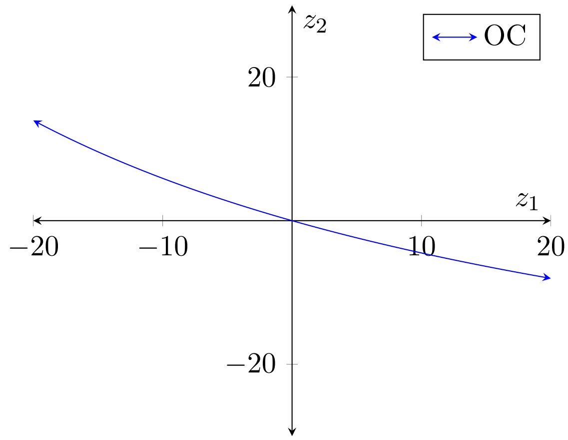
We solve the first of these equations for p which yields

$$p = \frac{(1 + \beta)z_1 + \beta\omega_1}{\omega_2}$$

We then plug this in to the resource constraint written in terms of excess supply: $z_1 = -pz_2$. Solving in terms of z_2 yields the following offer curve.

$$z_2 = \frac{-\omega_2 z_1}{(1 + \beta)z_1 + \beta\omega_1}$$
$$= \frac{-75z_1}{1.98z_1 + 147}$$

Which graphically is



Question 4. Overlapping Generations. 2 period lives. 1 commodity per period, $l = 1$. Stationary endowments:

$$\omega_0^1 = 2 > 0 \text{ for } t = 0$$
$$(\omega_t^t, \omega_t^{t+1}) = (2, 2) > 0 \text{ for } t = 1, 2, ...$$

Stationary preferences:

$$u_0(x_0^1) = 4 \log x_0^1 \text{ for } t = 0$$
$$u_t(x_t^t, x_t^{t+1}) = \log x_t^t + 4 \log x_t^{t+1} \text{ for } t = 1, 2, ...$$

Taxation:

$$m_0^1 = 10 \qquad m_t^s = 0 \text{ otherwise}$$

Goods price of money is $p^m \geq 0$.

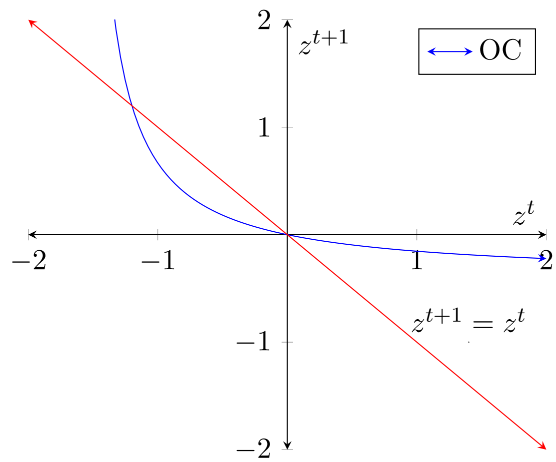
Derive the offer curve in excess demand space $(x_t^t - \omega_t^t, x_t^{t+1} - \omega_t^{t+1})$ for Mr. $t \geq 1$. Analyze the global dynamics.

Be precise. Find steady-state equilibria. Describe all possible paths. Include in your answer: hyperinflation, hyperdeflation, bursting bubbles, non-bursting bubbles.

Solution: We were asked to derive the offer curve (OC) in terms of Mr.

t's excess demands. Let $z^t = (x_t^t - \omega_t^t)$ and $z^{t+1} = (x_t^{t+1} - \omega_t^{t+1})$. The first order conditions yield the optimality condition $\frac{p^t}{p^{t+1}} = \frac{x_t^{t+1}}{4x_t^t}$. Plugging into the budget constraint yields the following OC, lying in quadrants 2 and 4:

$$z^{t+1} = \frac{-2z^t}{8 + 5z^t}$$

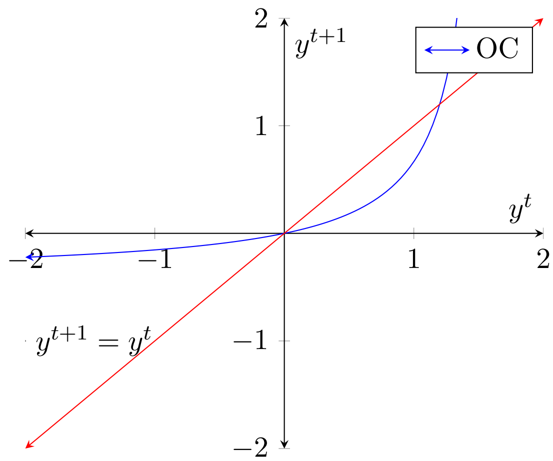


The reflected OC, which is more familiar in economic dynamics, lies in quadrants 1 and 3. (We will focus on quadrant 1 when analyzing the dynamics.) Let $y_t^t =$ excess demand by Mr $(t-1)$ for goods in period $t =$ excess supply of Mr. t for goods in period t.

$$y^t = (x_{t-1}^t - \omega_{t-1}^t) = (\omega_t^t - x_t^t)$$
$$\frac{y^{t+1}}{y^t} = \frac{p^t}{p^{t+1}} = R^t = (1 + r^t)$$

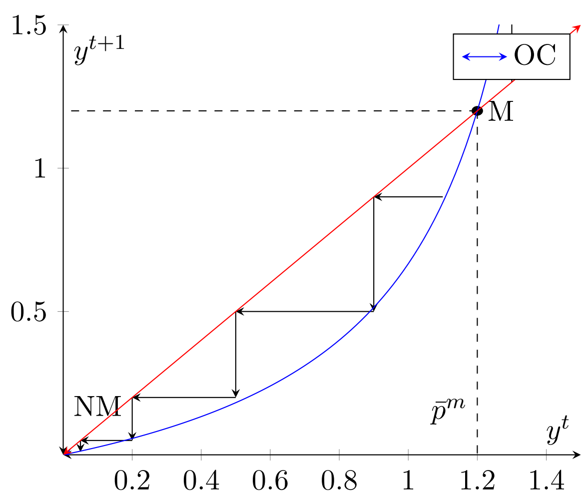
where p^t and p^{t+1} are present prices, R^t is the interest factor, and r^t is the interest rate. The reflected OC is given by

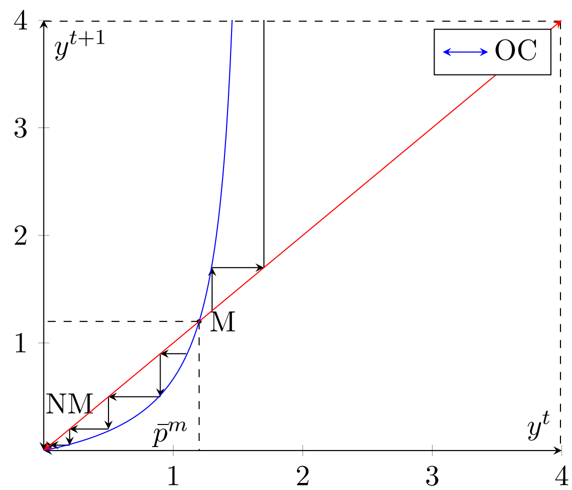
$$y^{t+1} = \frac{2y^t}{8 - 5y^t}$$



We redraw the reflected OC below solely in quadrant 1 as our phase diagram. We are in the Samuelson case, since

$$\frac{\frac{\partial u_t(\omega_t^t, \omega_t^{t+1})}{\partial x_t^t}}{\frac{\partial u_t(\omega_t^t, \omega_t^{t+1})}{\partial x_t^{t+1}}} = 1 + r < 1, \text{ i.e. } r < 0$$





4
y^{t+1}
3
2
1
OC
M
NM
$\bar{p}^m$
y^t
1
2
3
4

There are 2 stationary states.

(a) The non-monetary (non-PO) steady state with $y^t = y^{t+1} = 0$ (autarky), labelled NM.

(b) The monetary (PO) steady state with $y^t = y^{t+1} = 10\bar{p}^m = \frac{6}{5}$ and $\bar{p}^m = \frac{6}{5} \times \frac{1}{10} = \frac{3}{25}$, labelled M.

The non-monetary steady state is locally stable. The monetary steady state is unstable.

If $0 < p^m < \bar{p}^m$, the economy is inflationary. The current goods price of money tends asymptotically to zero. The money bubble fades away, but it does not burst.

If $p^m > \bar{p}^m$, the economy is deflationary. The current goods price of money grows so that in finite time demand for goods exceeds supply, so the deflationary bubble must burst (in finite time) since $x_t^t + x_t^{t+1} > \omega_t^t + \omega_t^{t+1}$, i.e. the demand for goods excess supply, see the phase diagram. Outside the 4 x 4 box competitive equilibrium cannot obtain. The hyper-deflationary path does not satisfy long-run perfect foresight. The bubble must burst in finite time.

Question 5. Inter temporal economies. Perfect borrowing and lending in terms of money. One commodity per period. Two periods. Present prices $p^1 = 1, p^2, p^{m1}, p^{m2}$. Perfect spot markets. No outside money. Rational expectations.

(a) Define equilibrium

(b) Show that, in equilibrium, we have $p^{m1} = p^{m2} = p^m \geq 0$.

(c) Describe fully the economics of (b).

(d) Show that if $p^m > 0$, then the equilibrium allocation to the money-and-spots economy is the same as the equilibrium allocation to the forwards-market economy.

(e) What happens if $p^m = 0$?

Solution:

(a) An equilibrium in this inter temporal economy with money is a price system $\{p^1 = 1, p^2, p^{m1}, p^{m2}\}$, allocations, $\{x_h^1, x_h^2\}\,\forall\, h$ and gross money holdings, $\{x_h^{m,1}, x_h^{m,2}\}\,\forall\, h$. Such that, taking prices as given, allocations and money holdings solve the utility maximization problem of consumer h. And we have goods market clearing: $\sum_h x_h^t = \sum_h \omega_h^t$ for $t = 1, 2$. And money market clearing: $\sum_h x_h^{m,t} = 0$ for $t = 1, 2$

(b) Assume that $p^{m1} \neq p^{m2}$, without loss of generality let $p^{m2} > p^{m1}$. Then combining the budget constraints of Mr. h gives us

$$x_h^1 + p^2 x_h^2 \leq \omega_1 + p^2\omega_2 - p^{m1}x_h^{m,1} - p^{m2}x_h^{m,2}$$

Set $x_h^{m,1} = -x_h^{m,2}$, and let Mr. h pick $x_h^{m,2}$ arbitrarily large, financed through $x_h^{m,1}$. Then he can pick arbitrarily large consumption, and all agents would do this. Thus we can't have goods markets clearing, and so this can't hold in equilibrium. So $p^{m1} = p^{m2} = p^m$.

(c) Each agent would take advantage of the arbitrage offered by the prices of money in different periods. Each agent will want to buy as much money during the period in which it is cheaper, and then sell it off when it is worth more. So this couldn't hold in a rational expectation equilibrium.

(d) The problem of Mr. h in the forward market can be written as

$$\begin{aligned} &\max_{c_h^1, c_h^2} U_h(c_h^1, c_h^2) \\ &\text{s.t. } c_h^1 + p^2 c_h^2 \leq \omega_1 + p^2\omega_2 \end{aligned}$$

Where goods markets must clear.
And the problem of Mr. h in the money and spots market can be written as

$$\begin{aligned} &\max_{c_h^1, c_h^2} U_h(c_h^1, c_h^2) \\ \text{s.t. } & c_h^1 + p^m x_h^1 \leq \omega_h^1 \\ & p^2 c_h^2 + p^m x_h^2 \leq p^2 \omega_h^2 \\ & x_h^1 + x_h^2 = 0 \end{aligned}$$

With goods and money markets clearing.
Solving the first constraint of Mr. h in the money and spots markets leads to

$$x_h^1 = \frac{x_h^1 - \omega_h^1}{p^m}$$

Which using the third constraint and plugging into the second implies that

$$c_h^1 + p^2 c_h^2 \leq \omega_1 + p^2 \omega_2$$

And since goods markets must clear, we get that the two types of markets lead to equivalent solutions.

(e) If $p^m = 0$, then we can no longer solve the first constraint for x_h^1. Intuitively there is no way to transfer wealth through time as there would be when either money is valued or we have a futures market.