

Question 1. Overlapping Generations

Consider an economy that consists of overlapping generations of two-period lived people. There is one consumption good. One young agent is born at each date $t \geq 1$, receives stationary endowment $(\omega_t^t, \omega_t^{t+1}) = (\omega_1, \omega_2)$, and ranks consumption plans according to $u(c_t^t, c_t^{t+1}) = \log c_t^t + \log c_t^{t+1}$. Consumption c_s^t is the amount consumed in period t by an agent born at date s . Assume that $\omega_1 > \omega_2$. An initial old agent is endowed with $\omega_0^1 = \omega_2$ and ranks consumption according to $u(c_0^1) = \log c_0^1$.

- (a) Define and solve for the non-monetary competitive equilibrium.
- (b) Solve the optimization problem of a social planner who attaches equal weights on each generation.
- (c) Define the competitive equilibrium with fiat money. Show that in equilibrium it must be that $p^{t,m} = p^{t+1,m} = p^m \geq 0$, and $p^m(x_t^{t,m} + x_t^{t+1,m}) = 0$.
- (d) Solve for the stationary equilibrium such that money has value. Assume that $m_0^1 > 0$, $m_t^s = 0 \forall t \geq 1$.
- (e) Draw the offer curve and resource constraint in excess demand-excess supply space and describe the dynamics for the non-stationary equilibria.