Overlapping Generations II

Question 1. Basic OLG
Consider a first economy (Economy I) that consists of overlapping generations of two-period lived people. A constant number $N$ young people are born at each date $t \geq 0$, each with utility function $u(c^t_s, c^{t+1}_s) = \log(c^t_s) + \log(c^{t+1}_s)$, where $c^t_s$ is period $t$ consumption for an agent born in period $s$. For all dates $t \geq 1$, young people are endowed with $y > 0$ units of a non-storable consumption good when they are young and no endowment when they are old. In addition, there are $N$ old people at time $t = 0$, who are collectively endowed with $H$ units of unbacked fiat currency. Let $p_t$ be the nominal price level at period $t$, denominated in dollars per period $t$ goods.

a) Define and compute an equilibrium with valued fiat currency for this economy. Argue that it exists and is unique.

b) Could the government implement this CE allocation via tax-and-transfer policy if the initial old were not endowed with any fiat currency?

Question 2. Social Security
Now consider a second economy (Economy II) that is identical to Economy I except that Economy II has a Social Security system (i.e., a pay-as-you-go pension system run by the government). The Social Security system works as follows: At each date $t \geq 0$, the government taxes $\tau > 0$ units of period $t$ consumption goods away from each young person and redistributes $\tau$ units of the time $t$ consumption good to each old person alive.

a) Does Economy II posses a stationary equilibrium with valued fiat currency? Is it unique?

b) Describe the restrictions on the parameter $\tau$, if any, that are needed to ensure the existence of an equilibrium with valued fiat currency.

c) Describe how the value of currency or price level would vary across economies with differences in the size of the Social Security system (as measured by $\tau$).