There are two monies, Red(R), and Blue(B). The units are R$ and B$.

Calculate the exchange rate between R$ and B$ for each of the following tax and transfer systems (giving units in your answers):

(a) $\tau_R = (2, 1, 0), \tau_B = (5, 3, 12)$

(b) $\tau_R = (5, 4, 2), \tau_B = (1, 0, 0)$

(c) $\tau_R = (8, 2, 6), \tau_B = (4, 1, 5)$

(d) $\tau_R = (7, 2, 12), \tau_B = (6, 5, 2)$

(e) In (a) - (d), are your answers independent of endowments? Why?

Solution:

The general problem of Mr. $h$ is

$$\max_{x_h} u_h(x_h)$$

subject to

$$px_h + P^{mR} \tau_R^h + P^{mB} \tau_B^h = p\omega_h,$$

where $P^{mR}$ is the number of chocolates per R$. We take the sum over all $h$ and use the goods market clearing condition $\sum_h x_h = \sum_h \omega_h$, yielding

$$P^{mR} \sum_h \tau_R^h + P^{mB} \sum_h \tau_B^h = 0 \tag{1}$$

When well defined, we want to solve for the exchange rate between R$ and B$, $XR_{RB} = \frac{p^{mR}}{p^{mB}} = -\frac{\sum_h \tau_B^h}{\sum_h \tau_R^h}$. Measured as chocolates per R$ relative to chocolates per B$, $\frac{B_S}{R_S}$ are the units for the exchange rate.
(a) Neither tax system is balanced \((\sum_{h} \tau_{h}^{R} > 0 \text{ and } \sum_{h} \tau_{h}^{B} > 0)\), so we know that \(P^{mR} = P^{mB} = 0\). Hence the equilibrium exchange rate is indeterminate \((XR_{RB} = \frac{0}{0} \frac{B}{R})\).

(b) Neither tax system is balanced \((\sum_{h} \tau_{h}^{R} > 0 \text{ and } \sum_{h} \tau_{h}^{B} > 0)\), so we know that \(P^{mR} = P^{mB} = 0\). Hence the equilibrium exchange rate is indeterminate \((XR_{RB} = \frac{0}{0} \frac{B}{R})\).

(c) Neither tax system is balanced \((\sum_{h} \tau_{h}^{R} > 0 \text{ and } \sum_{h} \tau_{h}^{B} > 0)\), so we know that \(P^{mR} = P^{mB} = 0\). Hence the equilibrium exchange rate is indeterminate \((XR_{RB} = \frac{0}{0} \frac{B}{R})\).

(d) Neither tax system is balanced \((\sum_{h} \tau_{h}^{R} > 0 \text{ and } \sum_{h} \tau_{h}^{B} > 0)\), so we know that \(P^{mR} = P^{mB} = 0\). Hence the equilibrium exchange rate is indeterminate \((XR_{RB} = \frac{0}{0} \frac{B}{R})\).

(e) For all parts, we cannot determine the equilibrium exchange rate, irrespective of endowments. In more general cases, the exchange rate is determined in financial markets independent of the real economy.

2

Assume \(\omega = (\omega_{1}, \omega_{2}, \omega_{3}, \omega_{4}, \omega_{5}) = (150, 80, 75, 25, 10)\). For each of the following, calculate the set of equilibrium money prices, \(P_{m}\):

(a) \(\tau = (40, 15, 10, -10, -30)\)
(b) \(\tau = (45, 15, 0, -10, -50)\)
(c) \(\tau = (30, 20, -5, -10, -35)\)
(d) \(\tau = (4, 0, 0, -2, -2)\)
(e) \(\tau = (5, 5, 1, 0, -5)\)

Solution:

Mr. h’s general problem is

\[
\max_{x_{h}} u_{h}(x_{h})
\]

subject to

\[x_{h} + P_{m} \tau_{h} = \omega_{h}.\]
We want the equilibrium money prices, $P^m$, such that $x_h > 0$ for all $h$. Thus we are looking at

$$x_h = \omega_h - P^m \tau_h > 0$$

for all $h$.

$$\Rightarrow \omega_h > P^m \tau_h$$

for all $h$.

Note that for $\tau_h \leq 0$ this obviously holds since $\omega_h > 0$. Thus we only need to check the condition for those $h$ such that $\tau_h > 0$. Or we can write it as

$$P^m < \min_h \left[ \frac{\omega_h}{\max(0, \tau_h)} \right].$$

Noting that the price of money can be zero in equilibrium, we now calculate this value for each part in order to get the set of equilibrium money prices, which follow.

(a) The tax system is not balanced ($\sum \tau_h = 25$), hence the set of equilibrium prices of money must be $P^m = \{0\}$.

(b) $P^m \in [0, 3\frac{1}{2})$

(c) $P^m \in [0, 4)$

(d) $P^m \in [0, 37\frac{1}{2})$

(e) The tax system is not balanced ($\sum \tau_h = 6$), hence the set of equilibrium prices of money must be $P^m = \{0\}$.

3

Write a concise paragraph on each of the following topics:

(a) bank runs

(b) dis-intermediation

(c) price-level volatility

(d) bubbles

Solution:
(a) Bank Runs. Bank runs can be caused by bad results on bank loans and bank portfolios or failures at creditor banks. When a bank faces liquidity problems, it might default including partial or total suspension of convertibility. Deposit insurance makes deposits safe but causes severe moral hazard issues. Many bank runs are purely or largely panic based. Depositors, even those with no need for liquidity, may withdraw early leading to a panic based run. This is the result of asymmetric information; the bank does not know which depositors have liquidity needs. "Bank runs" is also a metaphor for other pervasive financial fragility. For example, it is an explanation of some of the premia on sovereign debt interest rates in the eurozone since the sovereigns do not print euros.

(b) Dis-intermediation. The result of regulations that weaken existing financial products and financial institutions. For example, Fed Regulation Q banning or controlling checking account interest rates, led to check writing mutual fund and money market accounts.

(c) Price-level volatility. Money is a bubble, often a stochastic bubble driven by "sunspots", i.e. extrinsic uncertainty. Absence of money illusion allows for sunspot-driven price-level volatility.

(d) Bubbles. The technical definition of "bubble" is an asset whose price is less than its present value, Samuelson money is an obvious bubble asset. Other bubbles are difficult to detect before they burst or even then.

4

Let there be two consumers, Mr. 1 and Mr. 2.

(a) In \((\tau_1, \tau_2)\) space, graph the set of balanced taxes \(F_{bal}\).

(b) Graph the set of bonafide taxes \(F_{bon}\).

(c) Let \(\omega = (3, 7)\). Graph the set of normalized bonafide taxes, \(F^n_{bon}\), i.e. the set of taxes consistent with \(P^m = 1\).

(d) Let \(\omega = (3, 7)\). Use the diagram in (c) to calculate the set of equilibrium money prices. Hint: Use absence of money illusion.

Solution:
a) For taxes to be balanced we must have that $\sum h \tau_h = 0$. In our case this means that we have $\tau_2 = -\tau_1$. So the graph of $F_{bal}$ is as follows a $-45^\circ$ line.

b) We know that taxes are balanced if and only if they are bonafide. Hence the graph of $F_{bon}$ is the same as the graph for $F_{bal}$ from part (a).

c) We set $\omega = (3, 7)$ and now look for the taxes, $(\tau_1, \tau_2)$, that are consistent with $P_m = 1$. We are thus looking for taxes such that $P_m = 1$ is a consistent price given that the equilibrium price interval is determined by $P_m < \min \left[ \frac{3}{\max(0, \tau_1)}, \frac{7}{\max(0, -\tau_2)} \right]$. Plugging in $\tau_1 = -\tau_2$ and considering taxes consistent with $P_m = 1$ we therefore need to solve

$$1 < \min \left[ \frac{3}{\max(0, \tau_1)}, \frac{7}{\max(0, -\tau_1)} \right]$$

We proceed by cases.

**Case 1:** If $\tau_1 = 0$. Then the constraint becomes $P_m < \infty$, and hence $(0, 0)$ is consistent with $P_m = 1$.

**Case 2:** If $\tau_1 > 0$ then $P_m$ is constrained by $\frac{3}{\tau_1}$. So this is consistent with $P_m = 1$ provided that $\tau_1 < 3$.

**Case 3:** If $\tau_1 < 0$ this implies that $P_m$ is constrained by $\frac{7}{-\tau_1} = \frac{7}{\tau_2}$. Thus for this section to be consistent with $P_m = 1$ it must be that $\tau_2 < 7$. 

5
We therefore depict the set $F_{bon}^n$ as

d) Demand $x_h$ depends on taxes only through the product $P^m \tau_h$. This is "absence of money illusion", which is theoretically and empirically valid. It is weaker than the "quantity theory of money", which is neither empirically nor theoretically justified.