Bank Runs, Deposit Insurance, and Liquidity

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Topic Overview

- Economic Role of Banks: transform illiquid assets into liquid liabilities
- Bank Run: depositors expect that the bank will fail → everyone wants to withdraw deposits, even those who would normally prefer to wait
  - The bank must liquidate assets at a loss, and is at risk of failure
  - In a widespread panic (where many banks fail), the monetary system is negatively impacted
- Model demonstrates:
  1. Banks issuing demand deposits can improve a competitive market by providing risk sharing among people who need to consume at different times
  2. Undesirable equilibrium potential – bank run
  3. Bank runs cause issues through recall of loans and termination of investment

Agenda

Initial Assumptions and Autarky

Introducing the Bank

The Bank Run Equilibrium

Methods to Prevent Bank Runs

Initial Assumptions

1. There exist three periods
   – T = 0, 1, 2 ; T= 0 represents the current state

2. There is a single good

3. There exist a continuum of agents with measure 1

4. Each agent is endowed with 1 unit of the good in their initial period (T= 0)

5. Agents can store their goods at zero cost

The Model: Asset Return

With an initial endowment of 1 unit, each agent can either opt to exercise the good in period 1 or period 2

$$
\begin{array}{ccc}
T=0 & T=1 & T=2 \\
-1 & \begin{cases} 0 \\ 1 \end{cases} & \begin{array}{c} R \\ 0 \end{array}
\end{array}
$$

The Model: Asset Return

With an initial endowment of 1 unit, each agent can either opt to exercise the good in period 1 or period 2

$$T = 0 \qquad\qquad T = 1 \qquad\qquad T = 2$$

$$-1 \qquad\qquad \begin{cases} 0 \\ 1 \end{cases} \qquad\qquad \boxed{\begin{matrix} R \\ 0 \end{matrix}}$$

The amount the consumer gets in period two, denoted by the return "R" or '0", is determined by the agent's behavior in period 1

The Model: Preferences

Our model will take on the assumption that people have different behaviors

- In period 0, all agents are identical, and do not know if they are patient / impatient
  - T = 0, 1, 2 ; T= 0 represents the current state
- In period 1, some agents become "patient" and others "impatient"
  - This will affect their decision making

$$\begin{cases} u(c_1) & \text{Function if the agent is impatient} \\ u(c_2) & \text{Function if the agent is patient} \end{cases}$$

- The probability of being impatient is λ for each agent in period 0

Autarky

- Utility of the impatient person in period 1: u(1)
- Utility of the patient person in period 2: u(R)
- Expected utility in period 0: $\lambda u(1) + (1-\lambda)u(R)$

- $1 < R$
  - "Insurance" against the liquidity shock is desirable

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Banking Economy

- Bank offers demand deposit contract (d_1, d_2)
- Agents
  - Make deposits in period 0
  - Either:
    - Withdraw in period 1 (d_1)
    - Withdraw in period 2 (d_2)
- Free-entry banking sector: (d_1, d_2) maximizes the depositor's expected utility

Defining Variables / Parameters

T = Time period

d_i = Demand for deposits in T = i

λ = Probability of impatient

$1-\lambda$ = Probability of patient

R = Returns from the second Period

γ = Constant relative risk aversion

τ = Tax to fund deposit insurance

V_i = Payoffs in T=i

c_i = consuption in T=i, equivalent to d_i

ω_i = fraction of withdrawals demanded in T = i

f_j = fraction of population that withdraws before agent j

ρ = ratio of withdrawals in T=1 and T=2

Optimal Deposit Contract

$$\max : \lambda u(d_1) + (1-\lambda)u(d_2)$$

$$s.t. \quad \underbrace{(1-\lambda)d_2}_{withdrawals\ in\ period\ 2} \leq \underbrace{(1\text{-}\lambda d_1)R}_{resources\ in\ period\ 2} \qquad (RC)$$

$$d_1 \leq d_2 \quad (IC)$$

Optimal Deposit Contract

Solving for eg d_1 and d_2

$$Method\ 1:\ MRS = MRT$$

$$\frac{MU_1}{MU_2} = \frac{P_1}{P_2}$$

Optimal Deposit Contract

Solving for eg d_1 and d_2

$$Method\ 1:\ MRS = MRT$$

$$\frac{MU_1}{MU_2} = \frac{P_1}{P_2}$$

$$U(\mathrm{d}_1, \mathrm{d}_2) = \lambda u(\mathrm{d}_1) + (1-\lambda)u(\mathrm{d}_2)$$

$$MU_1 = \frac{\partial U}{\partial d_1} = \lambda u'(d_1)$$

$$MU_2 = \frac{\partial U}{\partial d_2} = (1-\lambda)U(d_2)$$

Optimal Deposit Contract

$$\text{Solving for eg } \mathrm{d}_1 \text{ and } \mathrm{d}_2$$

$$Method\ 1:\ MRS = MRT$$

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$$\frac{P_1}{P_2} = \frac{\Delta d_1}{\Delta d_2} = \frac{\lambda}{1-\lambda}R$$

Optimal Deposit Contract

Solving for eg d_1 and d_2

$$Method\,1: MRS = MRT$$

$$U(\mathrm{d}_1, \mathrm{d}_2) = \lambda u(\mathrm{d}_1) + (1-\lambda)u(\mathrm{d}_2)$$

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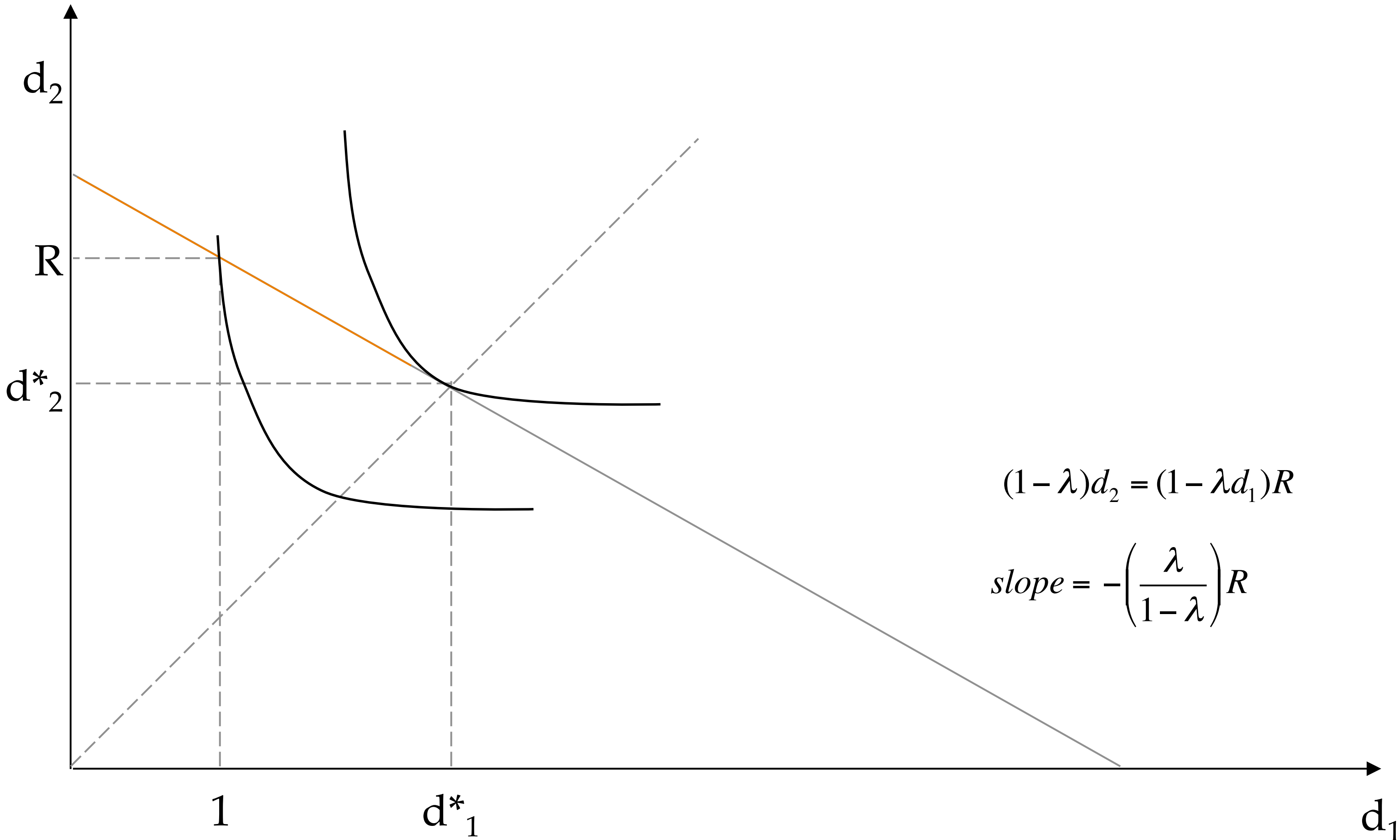
$$\frac{P_1}{P_2} = \frac{\Delta d_1}{\Delta d_2} = \frac{\lambda}{1-\lambda}R$$

$$\therefore \frac{u'(d_1)}{u'(d_2)}\left(\frac{\lambda}{1-\lambda}\right) = \left(\frac{\lambda}{1-\lambda}\right)R$$

$$\Rightarrow \frac{u'(d_1)}{u'(d_2)} = R \rightarrow Optimal\,Contract$$

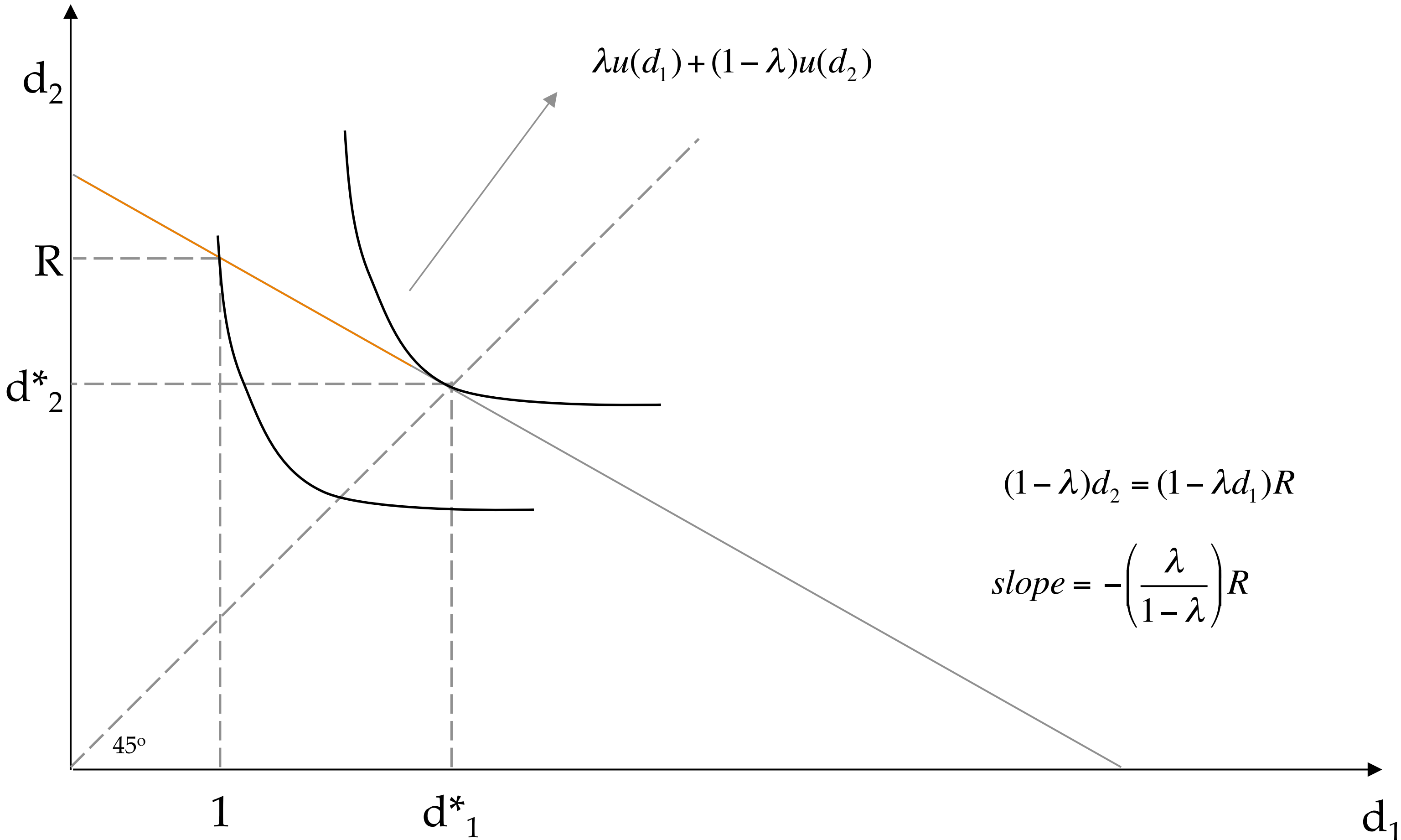
Optimal Deposit Contract



$$(1-\lambda)d_2 = (1-\lambda d_1)R$$

$$slope = -\left(\frac{\lambda}{1-\lambda}\right)R$$

Optimal Deposit Contract



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$$\underbrace{\frac{u'(d_1)}{u'(d_2)}\left(\frac{\lambda}{1-\lambda}\right)}_{MRS} = \underbrace{\left(\frac{\lambda}{1-\lambda}\right)R}_{MRT}$$

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Bank Runs

$$Let\ \ u(c) = \frac{c^{1-\gamma}}{1-\gamma}$$

$$0 < \gamma < 1$$

$$\gamma = 0 \Rightarrow No\ \ risk\ \ aversion$$

$$\gamma > 1 \Rightarrow More\ \ risk\ \ aversion$$

Bank Runs

$$Let\ \ u(c) = \frac{c^{1-\gamma}}{1-\gamma}$$

$$\gamma \ \text{ is a parameter } \in \left[0,1\right) \cup \left(1,\infty\right)$$

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$$\therefore R = \left(\frac{d_1}{d_2}\right)^{-\gamma} = \left(\frac{d_2}{d_1}\right)^{\gamma}$$

Bank Runs

$$Let\, u(c) = \frac{c^{1-\gamma}}{1-\gamma}$$

$$u'(c) = c^{-\gamma}$$

$$\therefore R = \left(\frac{d_1}{d_2}\right)^{-\gamma} = \left(\frac{d_2}{d_1}\right)^{\gamma}$$

$$Solving : (1-\lambda)d_1 R^{\frac{1}{\gamma}} = (1-\lambda d_1)R$$

$$\therefore d_1 = \frac{1}{\lambda + (1-\lambda)R^{\left(\frac{1}{\gamma}-1\right)}}$$

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Bank Runs

No run scenario

$r_1 = 1$

$V_1(f_n, 1) < V_2(f, 1) \forall 0 \leq f_j$

The bank would always have sufficient funds to pay agents r = 1 in T = 1

Not an optimal contract because it does not provide the impatient with insurance

Bank Runs

$$1 < c_1^{1*} < c_2^{2*} < R$$

$$\rho R > 1$$

$$1 < c_1^{1*}, R > c_2^{2*}, c_1^{1*} < c_2^{2*}$$

$$CRRA : U(c) = \frac{c^{1-\gamma} - 1}{1-\gamma}; \gamma > 1$$

$$c_2^{2*} = (\rho R)^{\frac{1}{\gamma}} c_1^{1*}$$

$$c_1^{1*} = \frac{R}{(1-\lambda)(\rho R)^{\frac{1}{\gamma}} + \lambda R}$$

$$c_2^{2*} = \frac{R(\rho R)^{\frac{1}{\gamma}}}{(1-\lambda)(\rho R)^{\frac{1}{\gamma}} + \lambda R}$$

$$\gamma > 1, \rho R > 1, c_2^{2*} > c_1^{1*}$$

Bank Runs

Proof:

$$Since: \ R > 1 \Rightarrow R \geq R^{\frac{1}{\gamma}}$$

$$\rho < 1 \Rightarrow 1 > \rho^{\frac{1}{\gamma}}, R > (\rho R)^{\frac{1}{\gamma}}$$

$$(1-\lambda)R > (1-\lambda)(\rho R)^{\frac{1}{\gamma}}$$

$$R > (1-\lambda)(\rho R)^{\frac{1}{\gamma}} + \lambda R$$

$$\Rightarrow c_1^{1*} > 1$$

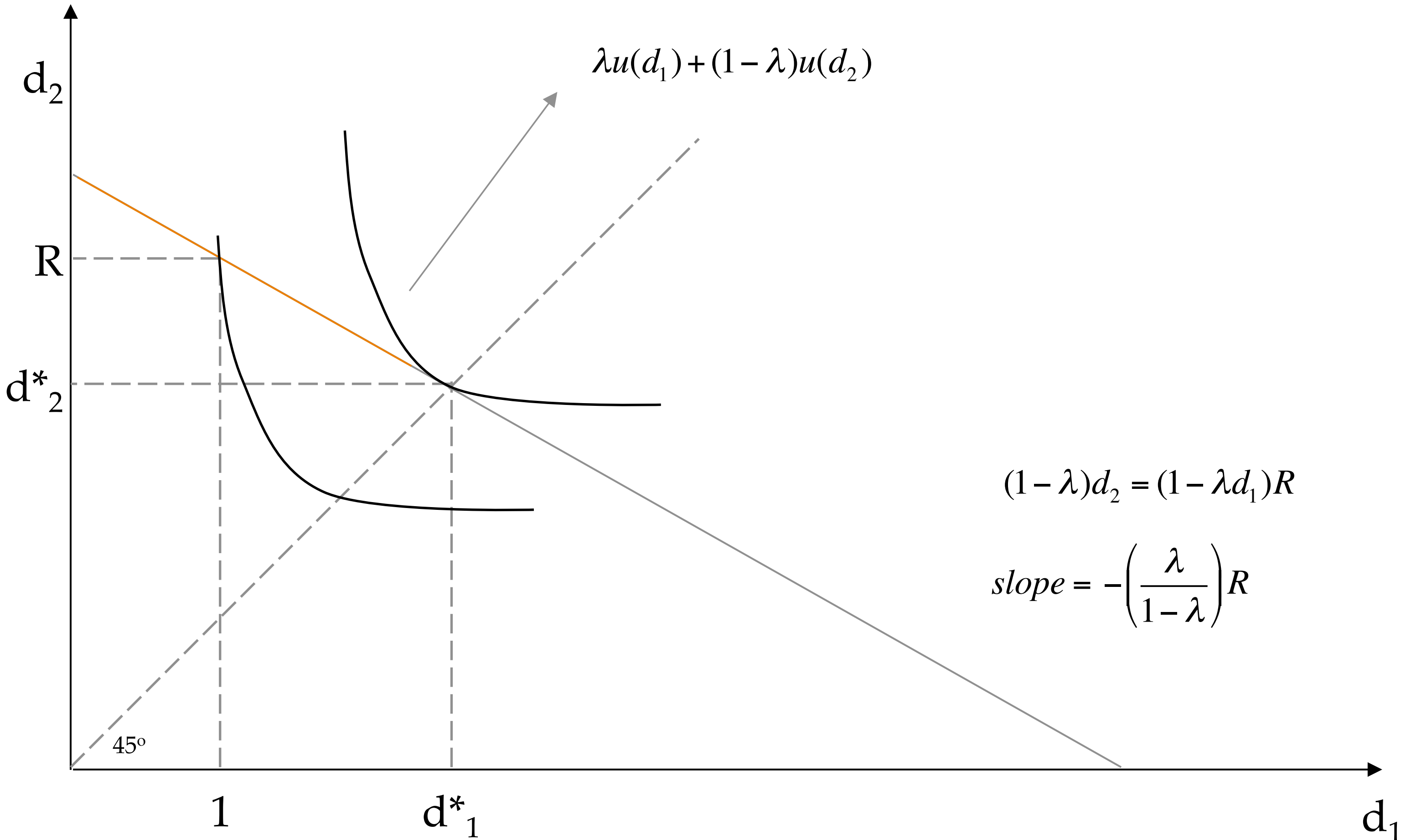
$$Substitute: \ c_1^{1*} > 1 \text{ into } RC$$

$$(1-\lambda)c_2^{2*} = R(1+\lambda c_1^{1*}) < R(1-\lambda)$$

$$c_2^{2*} < R$$

$$\therefore 1 < c_1^{1*} < c_2^{2*} < R$$

Optimal Deposit Contract



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$$\underbrace{\frac{u'(d_1)}{u'(d_2)}\left(\frac{\lambda}{1-\lambda}\right)}_{MRS} = \underbrace{\left(\frac{\lambda}{1-\lambda}\right)R}_{MRT}$$

Bank Runs

The less-than-optimal equilibrium where all agents become impatient

Deposit at $T = 0$, $c_1^1 = c_1^2 = c_2^1 = 0$

Payout in $T = 1$ is r_1 if the fraction of the withdrawals before the Agent J, (f_j) in $T = 1$

f = deposit withdrawals

V_1 = period 1 payoff per unit at T=1

f_j = # of withdrawer's deposits serviced before agent's as a fraction of demand deposits

$$V_1(f_j, r_1) = \begin{cases} r_1; f_j < \dfrac{1}{r_1} \\ 0; f_j > \dfrac{1}{r_1} \end{cases}$$

$$V_2(f, r_1) = \max\left\{R(1 - r_1 f/(1-f), 0\right\}$$

If $f > \dfrac{1}{r_1}$ the bank fails because they can't afford to pay people who withdraw in $T = 2$

* Everyone receives risky return with mean of 1

* All production interupted at $T = 1 \Rightarrow$ inefficient

Bank Runs

- A bank run has all agents withdraw their deposits at T = 1
  - If this is anticipated, all agents will prefer to withdraw at T = 1
  - The face value of deposits are larger than the liquidation value of the bank's assets
- The bank run equilibrium provides allocations that are worse for all agents than they would have obtained without the bank
- The "transformation" of illiquid assets into liquid assets is responsible
  - For the liquidity service provided by banks
  - For their susceptibility to runs

Bank Runs

- Agents deposit some of their wealth even if they anticipate a positive probability of a run
  - Provided that the probability is small enough
  - The good equilibrium dominates holding assets directly
- Runs happen if the selection between the bank run equilibrium and the good equilibrium depended on some commonly observed random variable in the economy
  - This could be a bad earnings report, a commonly observed run at some other bank, a negative government forecast, or even sunspots

Bank Runs

An optimal equilibrium for all agents

Only impatient people withdraw at T=1

Patient people withdraw at T=2

r_1 = payment in T=1

Set $r_1 = c_1$ = optimal consumption of Type 1 agent

$V_1(f_j, d_1) = c_1^{1*}$

$V_2(f, d_1) = c_2^{2*}$

$c_2^{2*} > c_1^{1*}$, $\rho R > 1$

$\therefore V_2(f, r_1) > V_1(f_j, r_1) \rightarrow$ Satisfies self selection constraint

Consumption of type 1 agent=$\omega_j V_1(f_j, d_1)$

Consumption of type 2 agent=$\omega_j V_1(f_j, d_1) + (1-\omega_j) r_2(f, d_1)$

Relative Risk Aversion

$$RRA = -C\frac{U''(C)}{U'(C)};\ U''(C) < 1 \rightarrow RRA$$

CRRA Utility Function: $\frac{C^{1-\gamma}}{1-\gamma}$; γ is a parameter $\in [0,1) \cup (1,\infty)$

If two individuals have different CRRA utility functions, the one with higher value of γ is considered to be more risk averse.

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$$U'(C) = C^{-\gamma}$$

$$U''(C) = -\gamma C^{-\gamma-1}$$

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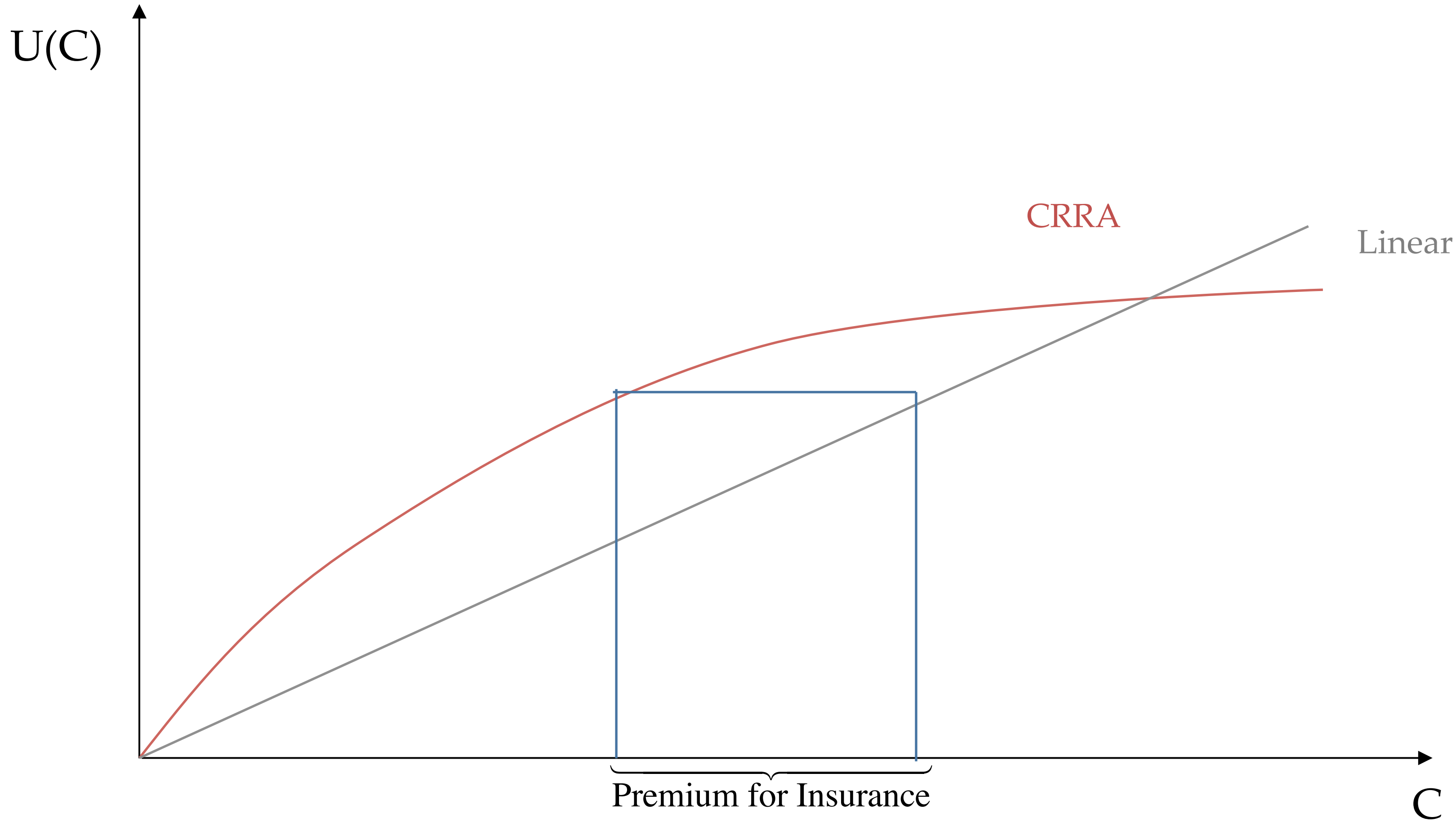
Relative Risk Aversion



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Tax on Withdrawing T= 1

$$Note: \text{If } d_1 > d_2 \text{ there is a run}$$

$$\tau \begin{cases} 0 & d_1 < d_2 \\ d_1 - 1 & d_1 > d_2 \end{cases};$$

$$V_1(d_1, d_2) \rightarrow pay\ offs\ \ T = 1$$

$$V_2(d_1, d_2) \rightarrow pay\ offs\ \ T = 2$$

Tax on Withdrawing T= 1

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$$V_1(d_1, d_2) \begin{cases} d_1 \\ d_1 - (d_1 - 1) = 1;\ d_2 < d_1 \end{cases}$$

τ is a reinvested into the bank and paid out in T=2

Tax on Withdrawing T= 1

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τ is a reinvested into the bank and paid out in T=2

$$V_2(d_1, d_2) \begin{cases} \dfrac{R(1-\lambda d_1)}{(1-\lambda)};\ \ d_1 < d_2 \\ \dfrac{R(1-\lambda d_1)}{(1-\lambda)} = R\ ;\ d_1 > d_2 \end{cases}$$

Optimal Contract / Role of Banks

- Provides insurance against being an impatient (Type 1) agent
- Optimal insurance is effective because all agents are satisfied with their consumption bundle → satisfies self-selection constraints
- Desirable Equilibrium
  - Banks provide liquidity so investors receive a return in the event that they must cash in before maturity
- Undesirable Equilibrium
  - Bank Run where the bank has insufficient funds available and must liquidate at a loss to return money to depositors

Demand Deposit Insurance

- Deposit Insurance does achieve optimal risk sharing by eliminating runs while preserving the bank's ability to transform assets
  - The bank is freed from dependence on the number of withdrawals
- Government Deposit Insurance through taxation
  - The government taxes those who withdraw in Period 1 depending on how many agents withdraw in Period 1 and how much they were promised
  - Unique from services provided by the bank → the government adjusts the tax after the period once it is possible to determine how many withdrawals are made in the period

Suspension of Convertibility

- Suspension of Convertibility Contracts can be used to prevent or stop bank runs
  - Type 2 agents are dissuaded from withdrawing early in anticipation of this policy
  - Regardless of other's withdrawals, the Type 2 agent maximizes utility by waiting until Period 2 to withdraw
    - This applies even if the Type 2 agent believes everyone else will act irrationally and try to withdraw early
- This is only an optimal contract when the normal number of withdrawals is known (not an optimal contract if the number of withdrawals varies)

Incentives

- Bank runs distort incentives because people panic
- Suspending convertibility when too many agents withdraw in Period 1 removes the incentive to run the bank
  - Ensures that participating in a bank run is never profitable
- Moral Hazard
  - If bank managers could select portfolios based on risk, there would be a trade off between optimal risk sharing and proper incentives for portfolio choice
  - If banks anticipate a bailout, they will take on high levels of interest rate risk

APPENDIX

Alternate Derivation

Method 2: Lagrange

$$\max\ \lambda u(d_1) + (1-\lambda)u(d_2);\ (1-\lambda)d_2 \le (1-\lambda d_1)R$$

$$\Rightarrow 0 \le (1-\lambda)d_2 - (1-\lambda d_1)R$$

$$\underbrace{\nabla U(d_1, d_2)}_{Curve} = C\nabla\Big((1-\lambda d_1)R - (1-\lambda)d_2\Big)$$

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$$\underbrace{\nabla U(d_1, d_2)}_{Curve} = C\nabla\Big((1-\lambda d_1)R - (1-\lambda)d_2\Big)$$

$$\lambda U'(d_1) = -C\lambda R$$

$$(1-\lambda)U'(d_2) = -C(1-\lambda)$$

Alternate Derivation

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$$\lambda U'(d_1) = -C\lambda R$$

$$(1-\lambda)U'(d_2) = -C(1-\lambda)$$

Dividing by each side, we obtain:

$$\frac{\lambda}{1-\lambda}\frac{U'(d_1)}{U'(d_2)} = \frac{\lambda R}{(1-\lambda)}$$