1 Overlapping Generations

2 period lives.
1 commodity per period, $l = 1$.
Stationary endowments:

$$\omega_0^1 = 2 > 0 \text{ for } t = 0$$
$$(\omega_t^t, \omega_t^{t+1}) = (2, 2) > 0 \text{ for } t = 1, 2, ...$$

Stationary preferences:

$$u_0(x_0^1) = 4 \log x_0^1 \text{ for } t = 0$$
$$u_t(x_t^t, x_t^{t+1}) = \log x_t^t + 4 \log x_t^{t+1} \text{ for } t = 1, 2, ...$$

Taxation:

$$m_0^1 = 10 \qquad m_t^s = 0 \text{ otherwise}$$

Goods price of money is $p^m \geq 0$.

Derive the offer curve in excess demand space $(x_t^t - \omega_t^t, x_t^{t+1} - \omega_t^{t+1})$ for Mr. $t \geq 1$. Analyze the global dynamics.

Be precise. Find steady-state equilibria. Describe all possible paths. Include in your answer: hyperinflation, hyperdeflation, bursting bubbles, non-bursting bubbles.

How does development in the OG endowment economy compare with development in the capital-and-money growth model?

Solution: We were asked to derive the offer curve (OC) in terms of Mr. t's excess demands. Let

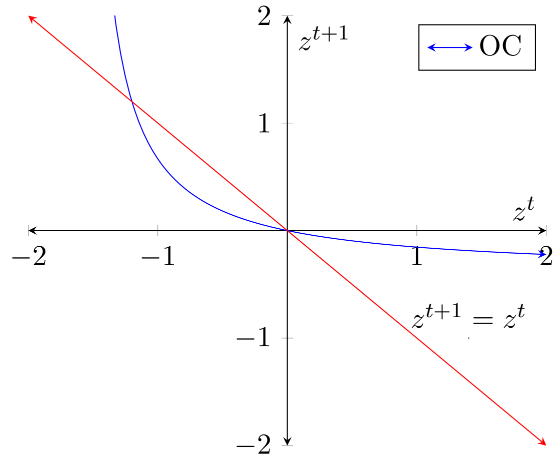
$$z^t = (x_t^t - \omega_t^t)$$

and

$$z^{t+1} = (x_t^{t+1} - \omega_t^{t+1})$$

This OC lies in quadrants 2 and 4 and is given by

$$z^{t+1} = \frac{-2z^t}{8 + 5z^t}$$



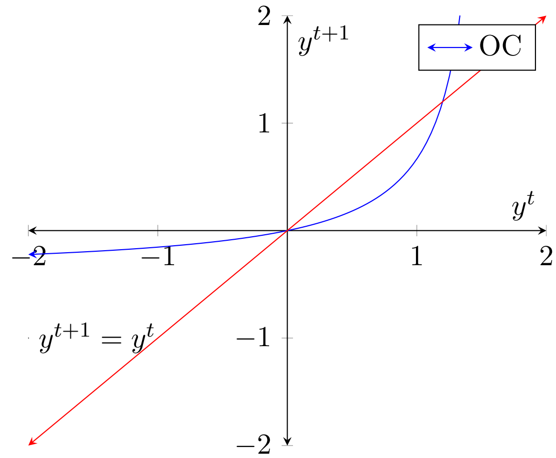
2
1
−1
−2
−2
−1
1
2
z^{t+1}
z^t
OC
$z^{t+1} = z^t$

The reflected OC, which is more familiar in economic dynamics, lies in quadrants 1 and 3 (We will focus on quadrant 1 when we do the dynamics). Let $y_t^t = \text{ excess demand } $ by Mr $(t-1)$ for goods in period $t = \text{ excess supply }$ of Mr. t for goods in period t.

$$y^t = (x_{t-1}^t - \omega_{t-1}^t) = (\omega_t^t - x_t^t)$$
$$\frac{y^{t+1}}{y^t} = \frac{p^t}{p^{t+1}} = R^t = (1 + r^t)$$

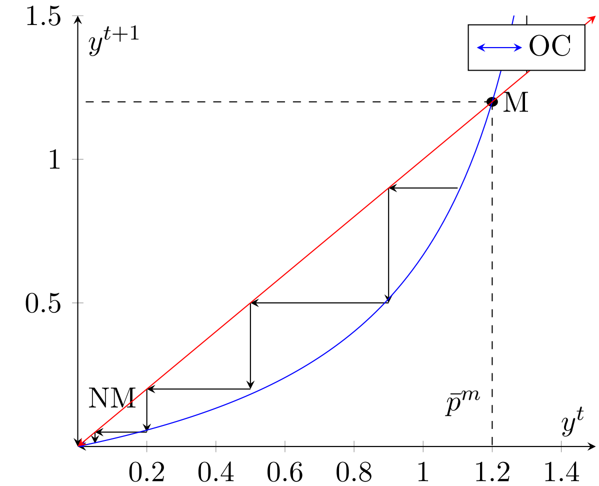
where p^t and p^{t+1} are present prices, R^t is the interest factor, and r^t is the interest rate.
The reflected OC is given by

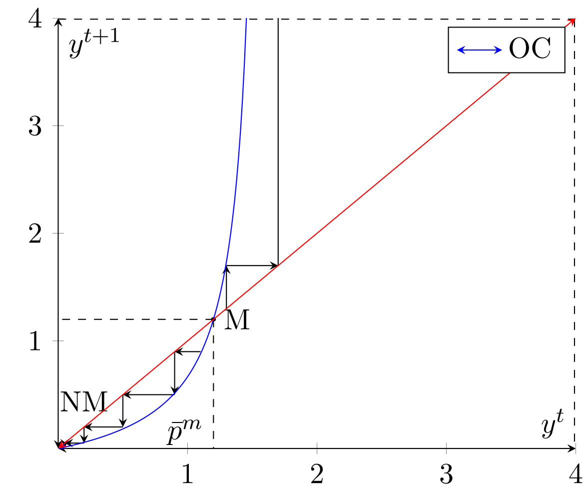
$$y^{t+1} = \frac{2y^t}{8 - 5y^t}$$


2
y^{t+1}
OC
1
y^t
-2
-1
1
2
$y^{t+1} = y^t$
-1
-2

We redraw the reflected OC below solely in quadrant 1 as our phase diagram. We are in the Samuelson case, since

$$\frac{\frac{\partial u_t(\omega_t^t, \omega_t^{t+1})}{\partial x_t^t}}{\frac{\partial u_t(\omega_t^t, \omega_t^{t+1})}{\partial x_t^{t+1}}} = 1 + r < 1, \text{ i.e. } r < 0$$


1.5
1
0.5
y^{t+1}
OC
M
NM
$\bar{p}^m$
y^t
0.2
0.4
0.6
0.8
1
1.2
1.4


4
3
2
1
y^{t+1}
OC
M
NM
$\bar{p}^m$
y^t
1
2
3
4

There are 2 stationary states.

(a) The non-monetary (non-PO) steady state with $y^t = y^{t+1} = 0$ (autarky) labelled NM.

(b) The monetary (PO) steady state with $y^t = y^{t+1} = 10\bar{p}^m = \frac{6}{5}$ and $\bar{p}^m = \frac{6}{5} \times \frac{1}{10} = \frac{3}{25}$, labelled M.

The non-monetary steady state is locally stable. The monetary steady state is unstable. If $0 < p^m < \bar{p}^m$, the economy is inflationary. The current goods price of money tends asymptotically to zero. The money bubble fades away, but it does not burst.
If $p^m > \bar{p}^m$, the economy is deflationary. The current goods price of money grows so that in finite time demand for goods exceeds supply, so the deflationary bubble must burst (in finite time) since $x_t^t + x_t^{t+1} > \omega_t^t + \omega_t^{t+1}$, i.e. the demand for goods excess supply, see the phase diagram. Outside the 4 x 4 box competitive equilibrium cannot obtain. The hyper-deflationary path does not satisfy long-run perfect foresight. The bubble must burst in finite time.

The dynamics are very similar:

(1) In the OG economy (pure-exchange, or in macro-speak, endowment economy) with Samuelson parameters:
The monetary (PO) steady state is unstable. If $p^m > \bar{p}^m$, the economy is in hyper-deflation. The monetary bubble must burst in finite time. If $p^m < \bar{p}^m$, the economy is inflationary. The monetary bubble fades away, but does not burst.

(2) In the money-and-growth model, there are also 2 steady states, 1 with positive real money holdings, the other with zero real money holdings. The non-monetary steady state is locally stable. The monetary steady state is unstable. Paths not tending to the monetary steady state either tend to the non-monetary steady state (on which the money bubble fades away but does not burst) or they reveal themselves in finite time to be disequilibrium paths (on which the money bubble bursts). On the latter paths the price of capital becomes zero in finite time, while the marginal product of capital is positive.