Economics 6130
Cornell University
Fall 2015
Macroeconomics, I - Part 2

Problem Set #4 Solution
December 2, 2015

Question 1

Consider the following economy:

$$K = K_1 + K_2$$
$$Y = C + Z_1 + 4Z_2 = K^{\frac{2}{3}} L^{\frac{1}{3}}$$
$$\dot{K}_1 = Z_1$$
$$\dot{K}_2 = Z_2$$
$$\frac{\dot{L}}{L} = .02$$
$$C = \frac{7Y}{10}$$

(a) What would a smart planner do about the allocation of investment between types of machines?

(b) Describe the momentary (static) equilibrium. What are the conditions on prices imposed by momentary equilibrium?

(c) Give the asset market clearing equations.

(d) Describe the trajectory of capitals and prices consistent with momentary equilibrium and efficiency of long-run development.

(e) Show that on other paths there are bubbles that burst in finite time.

Solution:

(a) Since the cost of capital investment Z_2 in terms of forgone consumption is four times that of Z_1, and since the two forms of capital are perfect substitutes for one another, it follows that the planner should optimally set $Z_2 = 0$ and use Z_1 for all investment.

(b) Markets clearing implies that

$$C = \frac{7Y}{10} = \frac{7(K_1 + K_2)^{\frac{2}{3}} L^{\frac{1}{3}}}{10} > 0$$

And

$$Z_1 + 4Z_1 = \frac{3Y}{10} = 3\frac{(K_1 + K_2)^{\frac{2}{3}} L^{\frac{1}{3}}}{10} > 0$$

Normalize the price of consumption to 1, then firms produce output and sell as either consumption or investment. This implies that

$$1 = \max\{p_1, \frac{p_2}{4}\}$$

Now profit maximization implies that the rental rates and wage are given by

$$r_i = \frac{2}{3}\Big(\frac{L}{K_1 + K_2}\Big)^{1/3} \text{ for } i = 1, 2$$
$$w = \frac{1}{3}\Big(\frac{K_1 + K_2}{L}\Big)^{2/3}$$

Now asset market clearing gives us that

$$\frac{\dot{p}_1}{p_1} + \frac{F_{K_1}}{p_1} = \frac{\dot{p}_2}{p_2} + \frac{F_{K_2}}{p_2}$$

Since marginal productivities are the same, and in static equilibrium we have that $\dot{p}_1 = \dot{p}_2 = 0$, this implies that we have $p_1 = p_2 = 1$ in equilibrium and hence $Z_2 = 0$ in equilibrium.

(c) The asset market clearing equation is

$$\frac{\dot{p}_1}{p_1} + \frac{F_{K_1}}{p_1} = \frac{\dot{p}_2}{p_2} + \frac{F_{K_2}}{p_2}$$

with

$$F_{K_1} = F_{K_2} = \frac{2}{3}\Big(\frac{L}{K_1 + K_2}\Big)^{1/3}$$

(d) Prices will be $p_1 = p_2 = 1$, and $\dot{p}_1 = \dot{p}_2 = 0$. This implies that $Z_2 = \dot{K}_2 = 0$, so we know that $\dot{K} = \dot{K}_1$. Now using that

$$\frac{\dot{K}}{L} = \dot{k} + nk$$

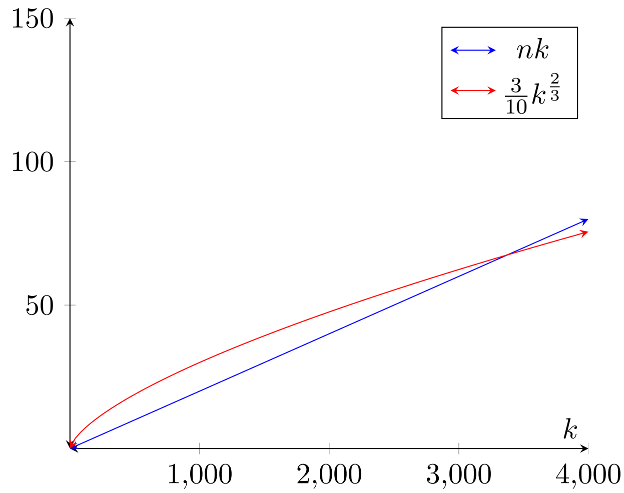
and that

$$\dot{K} = Z_1 = \frac{Y}{10} = \frac{3}{10}(K_1 + K_2)^{\frac{2}{3}} L^{\frac{1}{3}}$$

implies that

$$\dot{k} = \frac{3}{10} k^{\frac{2}{3}} - 0.02k$$

Which implies a value of $k^* = 15^3 = 3375$ and dynamics below such that any other value converges eventually to k^*



Note that $k = 0$ is also a long-run (unstable) equilibrium.

(e) Without loss of generality we consider the case of prices such that $p_1 < \frac{p_2}{4} = 1$. This implies that $\dot{p}_2 = 0$ and so asset market clearing becomes

$$\dot{p}_1(t) = \frac{2}{3}\Big(\frac{L(t)}{K_1(t) + K_2(t)}\Big)^{1/3}\big(\frac{p_1(t)}{4} - 1\big) < 0$$

So we have p_1 eventually falling at faster than a constant rate (because k tends to k^*), so in finite time T we will have that $p_1(T) = 0$ where $\dot{p}_1(t) = 0$ and the bubble bursts on or before date T.