Price Bubbles, the Risk-shifting Problem, and Financial Fragility

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Date: 30/09/2015
Major Topics to be Covered Today…

• The role of money & credit in asset price determination (Overarching topic)
• The Risk-shifting Problem (Expected Value using discrete random variables)
• Credit and interest rate determination (First-order derivatives)
• Financial risk and fragility
• Further extensions and discussions
Overview (1)

• In recent cases, intense asset price fluctuations are correlated to the credit expansion following financial liberalisations.

• After a country’s finance sector is liberalised, an expansion in credit and a surge in speculative activities can be reasonably expected.

• Some prominent cases: Japan in the early 1990s; the Scandinavian nations in the late 80’s; 1997 Asian Financial Crisis and the 2000 Dot-com bubble
Cases Overview…

<table>
<thead>
<tr>
<th>Country</th>
<th>Reason(s) for price bubble formation</th>
<th>Inducement(s) for bubble burst</th>
</tr>
</thead>
<tbody>
<tr>
<td>Japan in the late 1980s</td>
<td>Financial liberalisation throughout the 80s; Intention to support the US$</td>
<td>Policy reversal by the Bank of JPN (concerned with controlling inflation; tightened monetary policy)—effect on ir and bubble</td>
</tr>
<tr>
<td>Norway and Finland</td>
<td>N: The ratio of bank loans to nominal GDP soared; asset prices, I and C skyrocketed; F: an expansionary-budget-induced credit expansion and housing bubbles</td>
<td>N: Oil price collapse; F: tight monetary policy (ir, rrr); fall in trade with USSR</td>
</tr>
<tr>
<td>Mexico</td>
<td>Privatisation of banks; deregulation and the elimination of reserve requirements; lending boom</td>
<td>Political upheaval—assassination &amp; uprisings</td>
</tr>
</tbody>
</table>

Financial liberalisation & credit expansion generate bubbles; external and internal factors burst them.
Introducing/Reviewing Core Concepts

- **Price Bubbles**: Asset prices rise well above their fundamental/intrinsic values (Mishkin textbook definition)

- **Speculations**: the practice of engaging in risky financial transactions in order to profit from fluctuations in the market value of a tradable good such as a financial instrument, rather than attempting to profit from the underlying financial attributes embodied in the instrument such as capital gains, interest, or dividends. (Wikipedia definition)

- **Tight** monetary policies and Retrenchments—is there a role for monetary policies to prevent/inhibit negative bubbles? Japan, QE, and the Lost Decades?
In general, why would a price bubble burst?

• In slide 4 we have witnessed both external and internal factors that could trigger the burst of a price bubble. Allen and Gale (2004) summarised that the bubble bursts either because:
  
• 1) **returns on the assets are too low**, or
• 2) **credits are tightened by the central bank**
• …which brings us to the next question…
In theory, how would a price bubble collapse impact the banking sector?

- Banks hold **real assets** like land properties and **financial assets** like stocks and bonds, and they make loans to owners of such assets.
- When stock prices and housing prices plummet, banks’ assets shrink and the loan borrowers find it hard to pay back to the banks. However, banks’ liabilities are fixed. To meet deposit withdrawal requests, banks have to call in loans and liquidate their assets prematurely, which aggravate the existing problems—relates to **bank runs**, **vicious cycle** of falling prices, **MBS**, etc.
Then, how would a price bubble form in the first place?

• Consider a scenario when an investor borrows money to invest in financial assets:

• **Risk-shifting Problem**: investors obtain their funds from external sources. If fund providers cannot observe the characteristics (riskiness) of the investment, then the investor has the incentive to invest in riskier assets for greater expected returns, thereby **shifting the risk to the lender** of the fund and **bidding up the price** of the risky assets above the benchmark. (a concept developed in Allen and Gale (2000))

• The cost of default is fixed (**limited liability**); the expected return hinges more on the upper part of the return distribution.

• Another classical example of **Asymmetric information** and **Moral Hazard**.
To study how the risk-shifting problem affects price bubble formation, let us start from a simple model…

<table>
<thead>
<tr>
<th>Asset Type</th>
<th>Supply at t=1</th>
<th>Investment at t=1</th>
<th>Price of the asset at t=1</th>
<th>Payoff at t=2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Safe</td>
<td>Variable</td>
<td>1</td>
<td>1 per unit</td>
<td>1.5</td>
</tr>
<tr>
<td>Risky</td>
<td>Fixed, 1 unit</td>
<td>1</td>
<td>P per unit</td>
<td>R= 6, prob=25% R=1, prob=75% E(R)=2.25</td>
</tr>
</tbody>
</table>

- Assumptions: each investor has an initial wealth of 1 unit; he invests only with his own money; everybody is risk neutral and thus the marginal returns on the two assets should be equal.
- In this case, how should the risky asset be priced? Or, what is the value of P?
- Source:
\[ \frac{2.25}{P} = \frac{1.5}{1} = \text{the discount rate,} \]

• Hence P=1.5—the value of the asset is simply the discounted PV of the payoff and the discount rate is the OC of the investor.

• This is the classic definition of the fundamental/intrinsic value of the asset. Prices high above this benchmark will be called “bubbles” (end of case 1).
Case 2: leverage to invest in assets

• Here we will finally see an example of “risk shifting”

<table>
<thead>
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<th>Asset Type</th>
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</tr>
<tr>
<td>Risky</td>
<td>Fixed, 1 unit</td>
<td>1</td>
<td>P per unit</td>
<td>[R = 6, prob=25%](R=1, prob=75%)</td>
</tr>
</tbody>
</table>

• Investors borrow 1 unit of money from the bank to invest at 33.33%
• Can P=1.5 be the equilibrium price?
• Investors’ marginal return from safe asset: $1.5 - 1 \times (1 + 33.33\%) = 0.17$; the lender obtains 1.33

• Investors’ marginal return from risky asset: $0.25(\frac{1}{1.5} \times 6 - 1.33) + 0.75 \times 0 = 0.67$; the lender obtains $0.25 \times 1.33 + 0.75 \times 1 \times (\frac{1}{1.5}) = 0.83$

• Because the risky asset bestows greater expected return, investors put their money in them. However 0.5 in expected return was shifted from the lender to the borrower. This shift is caused by the higher risk of default (cat’s face)
• The lender won’t be happy about this, so a higher price is required for the risky asset in order to equate the expected return rate of the two assets and avoid the risk-shifting problem.

• In practice, the price of the risky asset, given this risk-shifting problem, will be bid up until the expected return of the risky asset is the same as the expected return of the safe asset (for the investor)

• So let’s redo the second question and see what’s the equilibrium price of the risky asset, here goes……
• Marginal return of risky assets = marginal return of safe assets

• \(0.25 \left( \frac{1}{p} \times 6 - 1.33 \right) + 0.75 \times 0 = 1.5 - 1.33\)

• Solve and find the value of \(P\): \(P=3\)

• A bubble above the benchmark of 1.5

• Therefore, the debt-financed investors are more willing to invest in assets priced above their fundamental. (end of case 2)
Summarising the points arisen from case 1 and 2…

• The amount of risk that is shifted depends on how risky the asset is;
• The greater the risk is, the greater the motivation to shift risk, and therefore the higher the price will be.

• Consider a third, more complicated case in which the expected return on the risky asset is a mean-preserving spread of the original returns (i.e. $E(R)$ is still 2.25, but the distribution of return is different from case 2)
Emulating the method that we’ve used in case 2, we can calculate the \( P \) in this case.

**Hint:** \( MR_{\text{risky}} = MR_{\text{safe}} \);

\( MR_{\text{risky}} = 0.25(\text{net return when not default}) + 0.75(\text{net return when default}) \)
• \(0.25 \left(\frac{1}{p} \times 9 - 1.33\right) + 0.75 \times 0 = 1.5 - 1.33\)

• \(p=4.5\)
Since the investors are indifferent between investing in the safe and risky asset, then the chance of default always exists.

Why the banks are willing to lend money to the investors?

For this to happen, banks’ expected marginal return must be greater than 1, so they must make sure that the majority of people will invest in safe assets.

Allen and Gale (2004) introduced a simple model to illustrate this point…
Going back to the 2\textsuperscript{nd} case when the equilibrium price of the risky asset is $P=3$……

• Assume there’s a fixed supply of risky asset of 1 unit
• Also suppose at the equilibrium the supply of risky asset meets demand
• And as usual, each investor has an initial endowment of 1 which they borrowed from the bank.
• Suppose there are a total of 10 borrowers in the economy
• When $P=3$, at equilibrium there are 3 people investing in risky assets and each of them gets $\frac{1}{3}$ unit of the risky asset.
• The remaining 7 investors will purchase safe assets
Calculating Banks’ expected return:

- \(0.3 \left(0.25 \times 1.33 + 0.75 \times \frac{1}{3} \times 1\right) + 0.7 \times 1.33 = 1.11 > 1\)

- In this case, when the ratio of safe investors to risky investors is 7:3, then the bank can reasonably expect a positive return when those investments mature.

- * Banks can also raise lending rates for the borrowers; it would be perfect if they can **distinguish the different types of borrowers and charge different rates**, but that is a different story from the one we’re interested in today 🙄 (cannot differentiate; high flat rate crowd out safe borrowers; adverse selection bla bla bla…)}
*Further thinking before concluding part 1…

- Is the “limited liability” hypothesis of the model always reasonable? The lenders is always able to pursue the debtor, or claim whatever they have, even if they can only do this in the future.
- If the borrowers understand that they will be pursued when a default occurs, will this knowledge serve as an inhibitor to keep them from engaging in too risky investment behaviours?
- If the lenders clearly state their pursuing policy before giving out the credit and exhibit a determination in carrying out the pursuit (“I mean what I’ve said”), I believe this will somewhat relieve the moral hazard of the investors.
Interest rates determination
(credit amount taken as exogenous)

• In the previous discussions, the quantity of credit supplied and the interest rate have been taken as exogenous

• the fundamental price of the risky asset is the discounted expected payoff:
  \[ P_F = \frac{E(R)}{r} \]

• In the following example, we still assume that the amount of credit supplied is exogenous (controlled by the central bank), and we will study how interest rates and asset price levels are determined.
• The Central Bank determines the amount of credit $B$ available to commercial banks (via rrr, dr, open market operations, QE, etc.)

• The banking sector is competitive; the number of banks and the number of borrowers are both normalised to 1. (1 bank lends to 1 borrower in the economy)

• Hence the investor gets an amount of $B$ from the bank

• The amount $B$ is invested in both safe assets (with an amount of $X$) and risky assets (with an amount of $P$). We have $X=B-P$. 
• The return rate of the safe asset can be denoted as $f'(B - P)$.

• Assume $F(X) = 3(B - P)^{0.5}$, then $f'(B - P) = 1.5(B - P)^{-0.5}$

• Provided that the loan market is perfectly competitive, then at equilibrium the interest rate on bank loans $r$ should equate the return rate of the safe asset $r = f'$ (also brainstorm the situations when $r > f'$ or $r < f'$)

<table>
<thead>
<tr>
<th>Asset Type</th>
<th>T=1</th>
<th>T=2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Safe</td>
<td>X is invested</td>
<td>F(X), or F(B-P) is obtained as return plus principal</td>
</tr>
<tr>
<td>Risky</td>
<td>$P$</td>
<td></td>
</tr>
</tbody>
</table>
In this competitive equilibrium, \( r = f' = 1.5(B - P)^{-0.5} \)

To calculate the maximum amount the investors are willing to pay for the risky assets, \( P \), can be calculated by equating the marginal return of risky assets with that of safe assets.

\[
0.25\left( \frac{1}{P} \times 6 - r \right) + 0.75 \times 0 = 0, \text{ plug in } r = f' = 1.5(B - P)^{-0.5}
\]

\[
P = 4(B - P)^{0.5}, \text{ and } P = 4\sqrt{B + 4} - 8
\]
• By controlling the amount of credit the CB controls the level of interest rates in a competitive loan market and set asset price levels.

• This is in contrast with the conditions in the standard asset pricing models in which both interest rates and credit amount are regarded as exogenous. In those cases, the fundamental price of a risky asset can be obtained simply with the formula \( P_F = \frac{E(R)}{r} \)

• (bring in the graph and the spotted line on p.244.)
What if the amount of credit B is uncertain as well?

- The analysis from slide 22-26 assumes that the quantity of credit, B, is still controllable by the central bank. What if this assumption no longer holds?
- In countries undergoing financial liberalisations, there are full of uncertainties about the level of B. So how should we investigate the effect of an uncertain B?
- In the following model, we extend the model to involve an extra period $t=0$. We also assume that the amount of credit B supplied at $t=1$ is uncertain.
Assume $F(X) = 3(B - P)^{0.5}$ and $f'(B - P) = 1.5(B - P)^{-0.5}$ still holds:

<table>
<thead>
<tr>
<th>Probability</th>
<th>$B_1$</th>
<th>$P_1$</th>
<th>$r_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>5</td>
<td>4</td>
<td>1.5</td>
</tr>
<tr>
<td>0.5</td>
<td>7</td>
<td>5.27</td>
<td>1.14</td>
</tr>
</tbody>
</table>

The asset pricing equation at $t=0$ is still $MR_{risk} = MR_{safe}$, and thus…
• \(0.5 \left( \frac{1}{P_0} \times P_1 - r_0 \right) + 0.5 \times 0 = 0 \) \——(1)

• \( r_0 = f' (B_0 - P_0) = 1.5 (B_0 - P_0)^{-0.5} \) \——(2)

• \( B_0 = 6 \) \——(3)

• From (1) (2) (3) we can get \( r_0 = 1.19 \) and \( P_0 = 4.42 \)

• Here the uncertainty is due to variations in credit supply (5 or 7). What effect will it have on \( P_0 \) and \( r_0 \) if the spread in credit supply distribution is greater?
Similarly, we adjust the previous table and change the value for the two B’s to 4 and 8, respectively…

<table>
<thead>
<tr>
<th>t=0</th>
<th>t=1</th>
<th>Remarks:</th>
</tr>
</thead>
</table>
| $B_0 = 6$; however the value for B at t=1 is uncertain | B=4 with Prob=0.5  
B=8 with Prob=0.5 | B=5 corresponds to default  
B=7 corresponds to success |

<table>
<thead>
<tr>
<th>Probability</th>
<th>$B_1$</th>
<th>$P_1$</th>
<th>$r_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>4</td>
<td>3.14</td>
<td>1.81</td>
</tr>
<tr>
<td>0.5</td>
<td>8</td>
<td>5.86</td>
<td>1.03</td>
</tr>
</tbody>
</table>

This is a mean-preserving adjustment of the credit amount B. Now a greater fluctuation in B implies greater financial uncertainty. Let’s see how this will affect $P_0$ and $r_0$. (4.608 and 1.272)
Compare the two cases:

• In the first case, $B=5$ or $7$, and the corresponding values for $P_0$ and $r_0$ are 4.42 and 1.19 ($t=1$ and $t=2$)
• In the second case, $B=4$ or $8$, and the corresponding values for $P_0$ and $r_0$ are 4.61 and 1.27 ($t=0$, $t=1$ and $t=2$)
• A more uncertain credit supply will lead to greater price bubbles.
• During financial liberalisations and credit expansions, more periods will be added into the investment cycle and it is fairly possible for the bubble to become very large (that’s why a abrupt monetary policy reversal might burst the bubbles)
Concluding part 1 and 2…

- What determines risky asset’s price at $t=0$?
- Expectation about aggregate credit supply at $t=1$
- What determines risky asset’s price at $t=1$?
- Concerns for risk shifting and the endeavour to equate $MR_{\text{risky}} = MR_{\text{safe}}$
- If credit supply goes up, then asset prices will rise up until the investor is indifferent between investing in safe or risky assets ($MR_{\text{risky}} = MR_{\text{safe}}$), provided that he is risk neutral
- If credit supply goes down, then asset prices will be low and investors will be prone to invest in risky assets for greater expected return. Then there’ll be a greater chance of default and risk-shifting problems will reoccur.
The importance of expectations in price bubble formation and investor’s decision-making

- Rational Expectations: expectation is identical to the optimal forecast, using all information available (Mishkin’s textbook definition of RE)
- The prospect of credit expansion is already taken into account, when the borrower is making his (1) borrowing decisions and (2) how much to pay for the risky asset.
- If credit expansion fails to meet anticipation, then P won’t be high enough to deter people from investing in risky assets, then the chance of default is higher and the borrowers may not be able to repay their loans. (Note here…)

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Supplementary Materials in case the previous materials are not interesting enough……

• The reason for supplementing these materials is that they directly correlate with the content on bank run that we had covered in the previous lectures.

• Using Allen and Gale (1998, 2004) model to discuss banking industry’s role in optimal risk sharing and devising optimal deposit contracts.

• In the following model, we assume depositors put their money into the bank which utilise the money to invest in asset market on behalf of the depositors.

• The bank engages in asset transformation and by pooling funds from a large number of depositors, the bank can offer insurance to consumers against their uncertain liquidity demands, giving depositors some of the benefits of the high-yielding risky asset without subjecting them to the full volatility of the asset market.
The Assumptions:

• 1 unit of endowment at t=0 for each depositor; the impatient withdraw only at t=1 and the patient withdraw only at t=2; depositor types revealed at t=1.

• Safe asset generates no extra returns (interest) E.g. 1 unit invested at date t will still worth 1 at date t+1. Risky asset transforms 1 unit invested at date t into R units at date t+2

<table>
<thead>
<tr>
<th>Asset Type</th>
<th>Shares held by bank</th>
<th>t=0</th>
<th>t=1; at this day a signal is released to inform the depositors the value of R to be realised at t=2 This is also the time those impatient depositors make their withdrawal decisions</th>
<th>t=2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Safe</td>
<td>y=1-x</td>
<td>1</td>
<td>1; safe assets mature early/are provide for the withdrawal of impatient depositors</td>
<td>1</td>
</tr>
</tbody>
</table>
| Risky      | x                   | 1   | Not yet matured; when they mature at t=2, they are provided for late withdrawals.   | R=R_H Prob.=π  
R=R_L Prob. =1−π  
Assume E(R)>1 |
Why do we call this model “optimal risk sharing”? 

- In this model, we think of banks as offering deposit contracts that maximise the expected utility of all depositors (both impatient and patient), while also to avoid bank runs (risk sharing).
- The amount of deposit that can be withdrawn at each date (t=1 or 2) is contingent on R. i.e. an impatient depositor who withdraws all his money at t=1 can get an amount of \( c_1(R) \); a patient one gets \( c_2(R) \) at t=2
- One extra point to notice: For the bank who made the portfolio investment decision, the risky asset return R is unknown at t=0, and therefore the portfolio choice is independent of R. However, the payment to depositors (t=1 or t=2), which occur after R is revealed at t=1, will depend on the value of R. That’s why we have \( c_1(R) \) and \( c_2(R) \) as functions of R
Suppose the probability that a depositor withdraws early is $\gamma$, and the probability that the depositor is patient and withdraws late is $1 - \gamma$. Hence for an individual investor his expected utility can be written as $\gamma U(c_1) + (1 - \gamma) U(c_2)$, and the optimal risk-sharing problem for every depositor can be written as:

$$\max E\{\gamma U(c_1(R)) + (1 - \gamma)U[c_2(R)]\},$$
subject to several constraints which are introduced in the next slide......
The four constraints placed on the utility maximisation function

- (1) The constraint on shares: the summation of the shares of risky asset and safe asset cannot exceed 1: \( x + y \leq 1 \)
- (2) The holding of safe asset must be sufficient to provide for the withdrawal of the impatient depositors at date \( t=1 \): \( \gamma c_1(R) \leq y \)
(3) The bank should hold strictly more than the amount above—it should ensure that its fund can meet the withdrawal demand from both types of the depositor. i.e. hold more than $\gamma c_1(R)$ and roll it over to $t=2$, in order to reduce the uncertainty of the patient depositors who withdraw late (a stronger version of (2))

$$\gamma c_1(R) + (1 - \gamma)c_2(R) \leq y + Rx$$

Or, written in a clearer format, $(1 - \gamma)c_2(R) \leq [y - \gamma c_1(R)] + Rx$

Patient depositors’ demand for withdrawal $\leq$ Leftover safe asset after the impatient guys are paid off

+ The total value (including return) of risky asset
• (4) The Incentive Compatibility Constraint (ICC): for every value of $R$, the late withdrawer must be at least as well off as the early withdrawer. Mathematically, we establish the following inequality $c_1(R) \leq c_2(R)$.

• ICC is extremely important since the late withdrawers have the option to imitate the early withdrawers to obtain $c_1(R)$ at date 1. They will refrain from early withdraw (a run on the bank) only if $c_1(R) \leq c_2(R)$ for every value of $R$.

• Mathematically we will find the ICC redundant and the first three constraints are suffice to produce a correct answer. But the ICC should be emphasised anyway.
• \( \max E\{\gamma U[c_1(R)] + (1 - \gamma)U[c_2(R)]\} \)
• s. t. (1) \( x + y \leq 1 \)
• (2) \( \gamma c_1(R) \leq y \)
• (3) \( \gamma c_1(R) + (1 - \gamma)c_2(R) \leq y + Rx \)
• (4) \( c_1(R) \leq c_2(R) \) (still point this out although mathematically redundant)
• Solution: (details can be found on Allen and Gale (1998))

• $c_1(R) = c_2(R) = y + Rx$, if $\frac{y}{\gamma} \geq \frac{Rx}{1-\gamma}$

• $c_1(R) = \frac{y}{\gamma}, c_2(R) = \frac{Rx}{1-\gamma}, c_1(R) < c_2(R)$, if $\frac{y}{\gamma} < \frac{Rx}{1-\gamma}$

• $x + y = 1$

• $E[U'(c_1(R))] = E[U'(c_2(R))R]$ This ensures equilibrium, like $\mu_1 = \mu_2$
- Substitute $L/2$ with $\frac{y}{\gamma}$; replace $R_{\text{bar}} = L/X$ with $\frac{(1-\gamma)y}{\gamma x}$.
- When $R=0$, both types choose to withdraw at $t=1$ and $c_1(0) = c_2(0) = y$.
- $(0, R_{\text{bar}}]$ impatient depositors get $\gamma y$ and the remaining $(1 - \gamma)y$ carried over to $t=2$ for patient depositors. Until $R_{\text{bar}}$ the optimal allocation involves carrying over some of the...
• Until $R^{\text{bar}}$ the optimal allocation involves carrying over some of the liquid (safe) asset to $t=2$ to supplement the low returns on the risky asset for late withdrawers.

• However for $[R^{\text{bar}}, +\infty)$, $R$ is so high that $c_2(R)$ begins to surpass $c_1(R)$.

• Now $c_1(R)$ is fixed at $\frac{\gamma}{\gamma}$, and $c_2(R) = \frac{Rx}{1-\gamma}$ gradually rise with $R$.

• After $R^{\text{bar}}$ the optimal allocation is simply (1) the impatient withdraw as much as possible at $t=1$, and the patient benefit from greater $R$. (no share)
Summary: in the optimal risk sharing example, deposit contracts are explicitly conditioned on $R$ and the bank knows the proportion of the two types of depositors.

However in reality contracts may not be explicitly conditioned on $R$, partly because $R$ is so unknown.
Further Extension

- So far we’ve talked about:

  (1) A mechanism of price bubble formation, i.e. risk shifting between the lender and the borrower.

  (2) Possible inducements for breaking a bubble

  (3) A case of optimal risk sharing among the bank, the patient and impatient depositors, and how to design deposit contract conditioned on the level of $R$ signalled at $t=1$ and to be realised at $t=2$
So what sort of financial crisis is the most dangerous for the economy?

- Debt-fuelled crises are the most dangerous and can spread to real economy
- Dot-com bubbles (didn’t lead to a huge recession) vs. 1929 stock market crash (Great Depression)
Bibliography

• Materials drawn upon: Chap 9 *Bubbles and Crises*, Understanding Financial Crises (Clarendon Lectures in Finance) Franklin Allen & Douglas Gale, Oxford University Press, 2009