• three periods: $T = 0, 1, 2$

• a single good

• a continuum of agents with measure 1

• Each agent is endowed with 1 unit of the good in period 0.
The Model: Asset Return

\[ T = 0 \quad T = 1 \quad T = 2 \]

\[ -1 \quad \begin{cases} 0 \\ 1 \end{cases} \quad R \quad 0 \]
The Model: Preferences

- In period 0, all agents are identical.
- In period 1, some agents become “patient” and others become “impatient”. (private information)
  \[
  \begin{cases}
  u(c_1) & \text{if impatient} \\
  u(c_2) & \text{if patient}
  \end{cases}
  \]
- The probability of being impatient is \( \lambda \) for each agent in period 0.
Autarky

- autarky:
  - utility of the impatient in period 1: $u(1)$
  - utility of the patient in period 2: $u(R)$
  - expected utility in period 0: $\lambda u(1) + (1 - \lambda) u(R)$

- $1 < R$
  - “insurance” against the liquidity shock is desirable.
Banking Economy

- Banks offers demand deposit contract \((d_1, d_2)\).
- Agents
  - make deposits in period 0.
  - withdraw \(d_1\) in period 1.
  - or withdraw \(d_2\) in period 2.
- free-entry banking sector: \((d_1, d_2)\) maximizes the depositor’s expected utility.
Optimal Deposit Contract

\[
\max_{d_1, d_2} \quad \lambda u(d_1) + (1 - \lambda) u(d_2)
\]

s.t. \hspace{1cm} (1 - \lambda) d_2 \leq (1 - \lambda d_1) R \hspace{1cm} (RC)

withdrawals in period 2 \hspace{1cm} resources in period 2

\[
d_1 \leq d_2 \hspace{1cm} (IC)
\]
Optimal Deposit Contract:

\[(1 - \lambda)d_2 = (1 - \lambda d_1)R\]

Slope: \[-\frac{\lambda}{1 - \lambda} R\]

45°
Optimal Deposit Contract:

\[ \lambda u(d_1) + (1 - \lambda)u(d_2) = \text{const} \]

slope = \(-\frac{\lambda}{1 - \lambda} \frac{u'(d_1^*)}{u'(d_2^*)}\)

\[ (1 - \lambda)d_2 = (1 - \lambda d_1)R \]

slope = \(-\frac{\lambda}{1 - \lambda} R\)

\[ \frac{\lambda}{1 - \lambda} \frac{u'(d_1^*)}{u'(d_2^*)} = \frac{\lambda}{1 - \lambda} R \]
What do banks do?

- \( u'(d_1^*) / u'(d_2^*) = R \)
- \( u'' < 0 \implies d_1^* < d_2^* \)
- CRRA: \( u(c) = \frac{c^{1-\gamma}}{1-\gamma} \)
  - \( u'(c) = c^{-\gamma} \implies u'(d_1) / u'(d_2) = (d_2 / d_1)^{\gamma} \)
  - if \( \gamma = 1 \implies d_1^* = 1, d_2^* = R \)
  - if \( \gamma > 1 \implies 1 < d_1^* < d_2^* < R \)
Why do bank runs occur?

- $\gamma > 1 \implies 1 < d_1^* < d_2^* < R$
- IC: $d_1 \leq d_2$
- Expectation: Only the impatient depositors withdraw in period 1.
- A patient depositor can
  \[
  \begin{cases} 
    \text{get } d_2^* & \text{if he withdraws in period 2} \\
    \text{get } d_1^* & \text{if he withdraws in period 1}
  \end{cases}
  \]
Why do bank runs occur?

- $\gamma > 1 \implies 1 < d_1^* < d_2^* < R$

- Expectation: All other depositors demand withdraw in period 1.

- A patient depositor can
  \[
  \begin{cases}
    \text{get } \textit{nothing} & \text{if he withdraws in period 2} \\
    \text{get } d_1^* \text{ w.p. } (1/d_1^*) & \text{if he withdraws in period 1}
  \end{cases}
  \]