Inside Money (continued)

\[ z_h^1 = -p^m m_h^1 = -L_h \]
\[ p^2 z_h^2 = p^m m_h^1 = L_h \]

If \( m_h^1 > 0 \), \( h \) is a lender.
If \( m_h^1 < 0 \), \( h \) is a borrower.

Optimal lending \( L_h = p^m m_h^1 \) in period one good terms depends on \( P^m \) and \( m_h^1 \) solely through the product: \( p^m m_h^1 \)

Let \( \hat{p}^m = \lambda p^m \),
Then \( \hat{L} = L \) if \( \hat{m}_h^1 = m_h^1 / \lambda \)

Combining equations:
\[ z_h^1(p^2) + p^2 z_h^2(p^2) = 0 \]
Outside Money

\( \tau_h \) is the money tax on Mr. \( h \)
\( \tau_h > 0 \) is tax \( \tau_h < 0 \) is a subsidy

Consumer Problem

\[
\max U_h(x_h^1, \ldots, x_h^i, \ldots x_h^t) \text{ subject to } \\
p x_h = p \omega_h - p^m \tau_h \text{ or } \\
p z_h = -p^m \tau_h, \ h = 1, \ldots, n
\]

Equilibrium \((p, p^m) \in \mathbb{R}_{++}^{2l} \times \mathbb{R}_+\)
\[
\sum_{1}^{n} z_h = 0 \\
x_h \in \mathbb{R}_{++}^{2l}, \ \omega_h \in \mathbb{R}_{++}^{2l}, \ z_h \in \mathbb{R}^{2l} \\
\tau \in \mathbb{R}^n \quad \tau = (\tau_1, \ldots, \tau_h, \ldots, \tau_n) \\
\text{Summing budget constraints over } h \text{ yeilds } \\
p \sum_{1}^{n} z_h = p^m \sum_{1}^{n} \tau_h
Definitions

\( \tau \) is said to be balanced if we have \( \sum_1^n \tau_h = 0 \). Ricardo for intertemporal interpretation.

\( \tau \) is said to be bonafide if there is a competitive equilibrium.

\((p, p^m)\) with \( p^m > 0 \)

From summed budget constraints we have

\[-p^m \sum_{1}^{n} \tau_h = 0\]

So either \( p^m = 0 \) or \( \sum_1^n \tau_h = 0 \) or both. An imbalanced \( \tau \) is not bonafide

Bonafide \( \Rightarrow \) Balanced
Balanced ⇔ Bonafide?

Tax adjusted Endowments

$$\tilde{\omega}_h = (\tilde{\omega}_h^1, ..., \tilde{\omega}_h^i, ..., \tilde{\omega}_h^l)$$
$$= (\omega_h^1 - p^m \tau_h, \tilde{\omega}_h^2, ..., \tilde{\omega}_h^i, ..., \tilde{\omega}_h^l)$$

For $p^m$ sufficiently small, $\tilde{\omega}_h \in \mathbb{R}_{++}^2$
Special Case

$l = 1$

$x_h \in \mathbb{R}^{2l}_{++}, \omega_h \in \mathbb{R}^{2l}_{++}, p \in \mathbb{R}^{2l}_{++}, p^m \in \mathbb{R}_+$

$p x_h = p \omega_h - p^m \tau_h, \ x_h = \omega_h - p^m \tau_h > 0$

let $P^m$ = goods price of money
$P = 1/P^m$ = general price level

Study the set of $P^m$ of equilibrium $P^m$:
Assume $\tau_h > 0$

Then we have
$0 \leq P^m < (\omega_h/\tau_h)$
Examples
\( \omega = (10, 8, 6, 4, 2) \)

1. \( \tau = (3, 3, 0, -3, -3) \)
   \[ \sum \tau_n = 6 - 6 = 0 \]
   Balanced
   \[ 3P^m < 10, \quad P^m < \frac{10}{3} \]
   \[ 3P^m < 8, \quad P^m < \frac{8}{3} \]
   \[ P^m = [0, \frac{8}{3}) \]

2. \( \tau = (3, 2, 0, -3, -3) \)
   \[ \sum \tau_n = 5 - 6 = -1 \neq 0 \]
   \[ P^m = 0 \]

3. \( \tau = (2, 0, 0, -1, -1) \)
   \[ \sum \tau_n = 2 - 2 = 0 \]
   \[ 2P^m < 10 \]
   \[ 0 \leq P^m < 5, \quad P^m = [0,5) \]

4. \( \tau = (1, 0, 0, 0, 0) \)
   \[ \sum \tau_n = 1 \neq 0 \]
   \[ P^m = 0 \]