Lecture 2

• Time (Intertemporal economics)
  • Future markets
  • Money markets & spot markets

• Uncertainty
  • Contingent claims markets
    • “Arrow-Debreu”
  • Securities Markets & Spot Markets
    • Arrow

• Reference
  • Arrow Paper in RES
    • Translated from CNRS
    • Translated from mimeo
Future Markets

- Time, \( t = 1, 2 \)
- Commodity \( l = 1 \) per period
- \( x_h^t > 0, \omega_h^t > 0 \) for \( t = 1, 2 \); \( h = 1, \ldots, n \)
- \( p_t \) price of commodity at time 1 to be delivered at time \( t \)
- Present prices
  - \( p^1 = 1 \)
  - \( p^2 = \frac{1}{1+r} = \frac{1}{R} \)
Futures Markets

\[ p^1 x_h^1 + p^2 x_h^2 = p^1 \omega_h^1 + p^2 \omega_h^2 \]

\[ p^1 (x_h^1 - \omega_h^1) + p^2 (x_h^2 - \omega_h^2) = 0 \]

\[ z_h^1 + \left( \frac{1}{1 + r} \right) z_h^2 = 0 \]

\[ h = 1, \ldots, n \]

\[ \sum_{h=1}^{n} z_h^t = 0 \quad t = 1, 2 \]

- Expectations are not up front because all trading is at time 1
- No re-trading at time \( t \)
  - By assumption
  or
  - By perfect foresight
• Futures market for commodities is closed
• Saving through lending money
• Dis-saving is through borrowing money
• Spot markets meet in $t = 1$ and $t = 2$
• Reasons money market is not “perfect”
  • DSGE (RBC)
  • DSGE with borrowing constraints
• Remarks on RCK
Inside Money

- AX traveler checks
- Here extended
- Isomorphic to Arrow article
- Irving Fisher
• Holdings of inside money
  • Purchase of inside money
    money = m^t_h \quad t = 1, 2; h = 1, \ldots, n
  • \sum_{h=1}^{n} m^t_h = 0 \quad t = 1, 2

• Outside money is created by the government and the banking system

• Present price of money
  \( p^{mt} \geq 0 \quad t = 1, 2 \)
Monetary Equilibrium

• Consumer Problem
  \[
  \max \quad V_h(x^1_h, x^2_h)
  \]
  subject to
  \[
  x^1_h + p^m_1 m^1_h = \omega^1_h
  \]
  \[
  p^2 x^2_h + p^m_2 m^2_h = p^2 \omega^2_h
  \]
  \[
  m^1_h + m^2_h = 0
  \]
  for \( h = 1, 2 \)

• Special Case

  • \( V_h(x^1_h, x^2_h) = U(x^1_h) + \beta U(x^2_h) \)

• Perfect Foresight about \( p^2 > 0 \) and \( p^m_2 > 0 \)

• Materials Balance
  \[
  \sum_h x^t_h = \sum_h \omega^t_h, \\
  \sum_{h=1}^t m^t_h = 0 \quad \text{for} \quad t = 1, 2
  \]

• Solve for \( p^2, p^m_1, \) and \( p^m_2 \)
Monetary Equilibrium (continued)

• Rewriting constraints
  • $z^1_h = -p^{m1}m^1_h$
  • $p^2 z^1_h = p^{m2}m^1_h$

• $z^1_h + p^2 z^1_h = (p^{m2} - p^{m1})m^1_h$

• Hence, $p^{m2} = p^{m1} = p^m \geq 0$

• Equilibrium allocation $x_h \in \mathbb{R}^{2n}_+$
is the same as for Future Market if $p^m > 0$
Uncertainty (isomorphic to intertemporal)

• See Arrow article
• 2 states of nature $s = \alpha, \beta$
• $h = 1, \ldots, n$ consumers
• Contingent commodity $x_h(s) > 0$
delivered only in state $s$
• Contingent endowments $\omega_h(s) > 0$
• Preferences
  $V_h(x_h(\alpha), x_h(\beta))$
  $= \pi(\alpha)U_h(x_h(\alpha)) + \pi(\beta)U_h(x_h(\beta))$
Contingent Claims (continued)

• Consumer Problem

\[ \max \pi(\alpha)U_h(x_h(\alpha)) + (1 - \pi(\alpha))U_h(x_h(\beta)) \]

Subject to

\[ p(\alpha)x_h(\alpha) + p(\beta)x_h(\beta) = p(\alpha)x_h(\alpha) + p(\beta)\omega_h(\beta) \]

Or

\[ p(\alpha)z_h(\alpha) + p(\beta)z(\beta) = 0 \]

Find \((p(\alpha), p(\beta))\) such that

• CP determines \(x_h(\alpha), x_h(\beta)\)

and materials balance

• \(\sum_h x_h(s) = \sum_h \omega_h(s) \text{ for } s = \alpha, \beta\)
• \( b_h(s) \) is the quantity bought of security \( s \)
• Security \( s \) pays 1 unit of account in state \( s \); otherwise, nothing
• \( p_b(s) \) is the price of security \( s \)
• \( p_b(\alpha) b_h(\alpha) + p_b(\beta) b_h(\beta) = 0 \)
• Purchase of security is financed by sale of other security (not necessary)
Arrow Securities (continued)

- CP

\[
\max \quad \pi(\alpha)U_h(x_h(\alpha)) + (1 - \pi(\alpha))U_h(x_h(\beta)) \quad \text{s.t.}
\]

1) \( p(\alpha)x_h(\alpha) = p(\alpha)\omega_h(\alpha) + b_h(\alpha) \)
2) \( p(\beta)x_h(\beta) = p(\beta)\omega_h(\beta) + b_h(\beta) \)
3) \( p_b(\alpha)b_h(\alpha) + p_b(\beta)b_h(\beta) = 0 \)

- Multiply (1) by \( p_b(\alpha) \) and (2) by \( p_b(\beta) \)

1) \( p_b(\alpha)p(\alpha)z_h(\alpha) = p_b(\alpha)b_h(\alpha) \)
2) \( p_b(\beta)p(\beta)z_h(\beta) = p_b(\beta)b_h(\beta) \)

But by 3) we have
\[
\hat{p}(\alpha)z_h(\alpha) + \hat{p}(\beta)z_h(\beta) = 0
\]
Where \( p_b(s)p(s) = \hat{p}(s) \) for \( s = \alpha, \beta \)
Arrow Securities (continued)

• CE is $(\hat{p}(\alpha), \hat{p}(\beta)) \in \mathbb{R}_+^{2n}$ in which

$$(x_h(\alpha), x_h(\beta)) \in \mathbb{R}_+^2$$ solves

PC for $h = 1, \ldots, n$

and

$$\sum_h z_h(s) = 0 \text{ for } s = \alpha, \beta$$
Conclusion

• Every contingent claims equilibrium allocation is also an Arrow securities equilibrium allocation

• Every AS equilibrium in which \( p_b(s) > 0 \) for \( s = \alpha, \beta \) is also CC equilibrium allocation

• Every FM equilibrium allocation is also an MM equilibrium allocation

• Every MM equilibrium allocation in which \( p^m > 0 \) is also an FM equilibrium allocation

• But interpretations of MM differ widely from interpretations of FM