Cornell University

Spring 2016

ECON 4905

Financial Fragility and the Macroeconomy

Prelim #1 – Answers

Monday, March 14, 2016, 202 Uris Hall

2:55PM to 4:10PM

Answer each of the 3 questions. The prelim is designed to take 60 minutes, but you may use the full 75 minutes.

Take advice from no source, neither animate nor inanimate.

Do not use calculators or any other electronic device.

Leave your books, electronic devices, and bulky items in the front of the room in a place chosen by the proctors.

1. (Outside) Money Taxation (20 minutes):

- 1 commodity, $l = 1$, chocolate measured in ounces
- 6 individuals, $h = 1, 2, ..., 6$
- taxes $\tau = (\tau_1, \tau_2, ..., \tau_6)$ measured in dollars ($)
- consumption $x = (x_1, x_2, ..., x_6)$ measured in ounces
- endowments $\omega = (\omega_1, \omega_2, ..., \omega_6) = (100, 90, 10, 10, 10, 10)$ measured in ounces
- $x_h = \omega_h - P^m \tau_h$ for h = 1, 2, ..., 6.

For each of (1a) through (1e), determine whether or not τ is balanced, whether or not τ is bonafide, solve for $\mathcal{P}^m$ (the set of equilibrium P^m) and for the allocation vector $x = (x_1, x_2, ..., x_6)$ as a function of P^m.

(1a) $\tau = (20, 20, -10, -10, -10, -10)$

(1b) $\tau = (100, 90, -20, -20, -20, -20)$

(1c) $\tau = (2, 2, -1, -1, -1, -1)$

(1d) $\tau = (0, 0, -5, -5, -5, -5)$

(1e) $\tau = (0, 0, 0, 0, 0, 0)$

(1f) In (1a) through (1e), which cases display multiple equilibrium allocations x? Which display multiple equilibrium prices P^m?

(1g) What light do your answers in (1a) through (1f) shed on financial fragility?

Solution:

(1a)

$$\sum_h \tau_h = 40 - 40 = 0$$
$$\tau \text{ balanced}$$
$$\tau \text{ bonafide}$$

$$\begin{aligned} 100 - 20P^m &> 0 \\ 20P^m &< 100 \\ P^m &< 5 \end{aligned}$$

$$\begin{aligned} 90 - 20P^m &> 0 \\ 90P^m &< 20 \\ P^m &< 9/2 < 5 \, * \end{aligned}$$

$$\mathcal{P}^m = [0, 9/2).$$

(1b)

$$\sum_h \tau_h = 190 - 80 = 110 \neq 0$$
$$\tau \text{ not balanced}$$
$$\tau \text{ not bonafide}$$

$$\mathcal{P}^m = \{0\}.$$

(1c)

$$\sum_h \tau_h = 4 - 4 = 0$$

$$\tau \text{ balanced}$$
$$\tau \text{ bonafide}$$

$$\begin{aligned} 100 - 2P^m &> 0 \\ 2P^m &< 100 \\ P^m &< 50 \end{aligned}$$

$$\begin{aligned} 90 - 2P^m &> 0 \\ 2P^m &< 90 \\ P^m &< 45 * \end{aligned}$$

$$\mathcal{P}^m = [0, 45).$$

(1d)

$$\sum_h \tau_h = 0 - 20 = -20 \neq 0$$

$$\tau \text{ not balanced}$$
$$\tau \text{ not bonafide}$$

$$\mathcal{P}^m = \{0\}.$$

(1e)

$$\sum_h \tau_h = 0$$

$$\tau \text{ balanced}$$
$$\tau \text{ bonafide}$$

$$\mathcal{P}^m = [0, \infty)$$

P^m indeterminate because there will be no money trades, no matter the price.

(1f)
- In (1a), there are in equilibrium multiple prices P^m and hence multiple allocations x.

- In (1b), $P^m = 0$ is the unique price and the allocation $x = \omega$ is unique.
- In (1c), in equilibrium there are multiple P^m and multiple allocations x.
- In (1d), equilibrium $P^m = 0$ and the equilibrium allocation $x = \omega$ are unique.
- In (1e), the equilibrium allocation $x = \omega$ is unique, but the price P^m is indeterminate.

(1g) In some cases, the equilibrium allocation is unique, but generally x depends on consumer beliefs about P^m. Fundamentals do not completely determine economic outcomes. Beliefs are important. This leads to financial fragility.

2. 2 currencies (R and B) (20 minutes):

$\omega = (\omega_1, \omega_2, \omega_3, \omega_4, \omega_5) = (5, 4, 3, 2, 1)$.
In each of (2a) through (2d) solve for the exchange rate e. Give the units of e.

(2a) $\tau^R = (2, 1, 1, 1, 0)$, $\tau^B = (2, 0, -1, -1, -1)$,

(2b) $\tau^R = (0, 0, 0, 0, 0)$, $\tau^B = (1, 0, 0, 0, -1)$,

(2c) $\tau^R = (-1, -1, -1, -1, -1)$, $\tau^B = (1, 1, 1, 1, 1)$,

(2d) $\tau^R = (2, -1, -1, -1, 0)$, $\tau^B = (10, 8, 0, 0, 0)$,

(2e) How are your answers in (2a) through (2d) changed if ω is replaced by $\omega' = (100{,}1{,}1{,}1{,}1)$? What is the economic argument behind this? What does it teach us?

Solution:

(2a)

$$\begin{aligned}
(2+1+1+1+0)P^R + (2+0-1-1-1)P^B &= 0 \\
5P^R - P^B &= 0 \\
5P^R &= P^B \\
\frac{P^B}{P^R} = 5 \qquad \frac{P^R}{P^B} &= \frac{1}{5}
\end{aligned}$$

1 blue dollar = 5 red dollars

(2b)

$$0P^R + 0P^B = 0$$
$$\frac{P^R}{P^B} = \frac{0}{0}$$

exchange rate is indeterminate

(2c)

$$-5P^R + 5P^B = 0$$
$$\frac{P^B}{P^R} = \frac{P^R}{P^R} = 1$$

1 blue dollar = 1 red dollar

(2d)

$$\begin{aligned}
(2-3)P^R + 18P^B &= 0 \\
-P^R + 18P^B &= 0 \\
18P^B &= P^R \\
\frac{P^R}{P^B} = 18, \qquad \frac{P^B}{P^R} &= \frac{1}{18}
\end{aligned}$$

1 red dollar = 18 blue dollars

(2e) The change from ω to ω' does not change the answers in (2a) – (2d). The reason is that in this simple model, exchange rates are determined solely in the foreign exchange market. It teaches us that financial matters can have profound effects on economic outcomes.

3. Inside Money (20 minutes):

1 commodity, $l = 1$; 2 periods, $t = 1, 2$; 2 consumers, h=1,2

$t = 1, 2, \quad h = 1, 2$

$u_h(x_h^1, x_h^2) = \log x_h^1 + \log x_h^2$

$\omega_1 = (\omega_1^1, \omega_1^2) = (2, 10)$

$\omega_2 = (\omega_2^1, \omega_2^2) = (10, 2)$

(3a) What is the equilibrium allocation $x = \big((x_1^1, x_1^2), (x_2^1, x_2^2)\big)$ when the money market is closed?

(3b) Is the closed-money-market equilibrium Pareto optimal?

(3c) Calculate the set of PO allocations in which $\lambda > 0$ is the welfare weight on Mr. 2.

(3d) What are the PO allocations corresponding to $\lambda = 0$, $\lambda = 1$, and $\lambda = \infty$?

(3e) Show that the $\lambda = 1$ PO allocation is *the* competitive equilibrium allocation when the money market is open.

Solution:

(3a) When the money market is closed there is no borrowing or lending, so no intertemporal trading. Hence in this simple model, the equilibrium allocation is $x = \omega$, autarky:

$$x = \big((2, 10), (10, 2)\big).$$

(3b) Autarky is not Pareto optimal. Since the log is a strictly increasing function, consumers prefer smoother consumption over time.

For example, perfectly smoothed consumption

$$x = \big((6, 6), (6, 6)\big)$$

is strictly preferred to autarky since

$$\log 6 + \log 6 > \log 2 + \log 10$$

and

$$\log 6 + \log 6 > \log 10 + \log 2.$$

(3c) Let $W = u_1 + \lambda u_2$ be social welfare, where $\lambda \geq 0$.

Let Mr 1 consume $(2 - x)$ in period 1 and $(10 - y)$ in period 2. Then Mr 2 consumes $(10 + x)$ in period 1 and $(2 + y)$ in period 2.

$$W = \log(2 - x) + \log(10 - y) + \lambda\big[\log(10 + x) + \log(2 + y)\big]$$

maximize with respect to x and y

$$\frac{\partial W}{\partial x} = \frac{-1}{2 - x} + \frac{\lambda}{10 + x} = 0$$

$$\frac{10 + x}{2 - x} = \lambda$$

$$\frac{\partial W}{\partial y} = \frac{-1}{10 - y} + \frac{\lambda}{2 + y} = 0$$

$$\frac{2 + y}{10 - y} = \lambda$$

(3d) Aggregate endowments:

$$(\omega_1^1 + \omega_2^1, \omega_1^2 + \omega_2^2) = (2 + 10, 10 + 2) = (12, 12)$$

How to allocate the aggregate endowments (12,12)?

Let x be the allocation of the period 1 good to Mr 1 and y be the allocation of the period 2 good to Mr 1. Then the respective allocation to Mr 2 are $(12 - x, 12 - y)$.

The planer's problem is to maximize the welfare function W, where W is given by

$$W = \log x + \log y + \lambda \big[\log(12 - x) + \log(12 - y) \big]$$

FOC:

$$\frac{\partial W}{\partial x} = \frac{1}{x} - \frac{\lambda}{12 - x} = 0,$$

$$\frac{\partial W}{\partial y} = \frac{1}{y} - \frac{\lambda}{12 - y} = 0.$$

$$(\lambda + 1)x = 12,$$

$$(\lambda + 1)y = 12.$$

When $\lambda = 1, x = y = 6$, and

$$(x_1^1, x_1^2) = (x_2^1, x_2^2) = (6, 6).$$

When $\lambda = 0, x = y = 12$, and

$$(x_1^1, x_1^2) = (12, 12), \ (x_2^1, x_2^2) = (0, 0).$$

When $\lambda = \infty, x = y = 12/\infty = 0$, and

$$(x_1^1, x_1^2) = (0, 0), \ (x_2^1, x_2^2) = (12, 12).$$

When $\lambda = 1$, the weights on the consumers are equal. The planner smooths utility and in this case smooths consumption. When $\lambda = 0$, 100% of the weight is on Mr 1. He is given the entire aggregate endowment. When $\lambda = \infty$, the entire weight is on Mr 2. He is given the entire aggregate endowment.

(3e) It might be simpler to analyze the equivalent futures market equilibrium. Assume that present commodity prices are: $p^1 = p^2 = 1$.

MAXIMAND $u_h = \log x_h^1 + \log x_h^2$

budget constraint is

$$
\begin{aligned}
x_h^1 + x_h^2 &= 12 \\
x_h^2 &= 12 - x_h^1 \\
u_h &= \log x_h^1 + \log(12 - x_h^1) \\
\frac{\partial u_h}{\partial x_h} &= \frac{1}{x_h^1} - \frac{1}{12 - x_h^1} = 0 \\
\frac{1}{x_h^1} &= \frac{1}{12 - x_h^1} \\
x_h^1 &= 6, x_h^2 = 6.
\end{aligned}
$$

So FM equilibrium is

$$(x_1^1, x_1^2) = (x_2^1, x_2^2) = (6, 6)$$

$6 + 6 = 12$ and $6 + 6 = 12$
materials balance.