1. (Outside) Money Taxation

- 1 commodity, $l = 1$, chocolate measured in ounces (oz.)
- 2 individuals, $h = 1, 2$
- taxes $\tau = (\tau_1, \tau_2)$ measured in dollars ($$)
- consumption $x = (x_1, x_2)$ measured in ounces
- endowments $\omega = (\omega_1, \omega_2) = (100, 25)$ measured in ounces
- $x_h = \omega_h - P^m \tau_h$

a. What are the units in which $P^m$ is measured?

((b) - (e)): In what follows, when is $\tau$ balanced or not, and when is $\tau$ bonafide or not? Solve for $P^m$, the set of equilibrium $P^m$.

b. $\tau = (5, -5)$

c. $\tau = (1, -7)$

d. $\tau = (1, -1)$

e. $\tau = (-1, -1)$

f. In (b) - (e), in which cases are there multiple equilibria? What are the lessons from this for macroeconomics?

Solution:

a. ounces per dollar, oz./$.

b. balanced, bonafide.

$x_1 = 100 - 5P^m > 0$, $P^m < 20$

$P^m = [0, 20)$.

c. not balanced, not bonafide.

$P^m = \{0\}$. 

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d. balanced. bonafide.
\[ x_1 = 100 - P^m > 0, \quad P^m < 100 \]
\[ P^m = [0, 100). \]

e. not balanced. not bonafide.
\[ P^m = \{0\}. \]

f. The price level is indeterminate in each of the balanced/bonafide cases: (b), (d), and (e). This emphasizes — particularly in money-finance economies — that the allocation of resources depends as much on beliefs as opposed to being determined solely by fundamentals (preferences and endowments).
2. Outside Money: 2 Currency Taxation

Same set-up as in (1.), but now 2 currencies: euro (€) and pound sterling (£). In each of the following solve for the exchange rate $e$. Give the units of $e$.

a. $\tau^e = (-1, -1), \tau^£ = (1, 1)$
b. $\tau^e = (1, -1), \tau^£ = (-5, 5)$
c. $\tau^e = (2, 1), \tau^£ = (1, -5)$

Solution:

a. 

\[ \sum_h \tau^e_h = -2e, \quad \sum_h \tau^£_h = 2£ \]

\[-2P^me + 2P^m£ = 0\]

\[P^me : \frac{€}{choc}, \quad P^m£ : \frac{£}{choc}, \quad \text{units}\]

\[\frac{P^me}{P^m£} = 1 = \frac{P^m£}{P^me}, \quad \text{exchange rates}\]

\[\frac{e}{£} = \frac{£}{e}, \quad \frac{P^m£}{P^me} = \frac{P^me}{P^m£}\]

1 euro = 1 pound sterling

$e = 1$

b. 

\[\sum_h \tau^e_h = 0, \quad \sum_h \tau^£_h = 0\]

\[0P^me + 0P^m£ = 0\]

$e$ is indeterminate

c. 

\[\sum_h \tau^e_h = 3e, \quad \sum_h \tau^£_h = -4£\]

\[3P^me - 4P^m£ = 0\]

\[3P^me = 4P^m£\]

\[\frac{P^me}{P^m£} = \frac{4}{3}\]

\[\frac{P^e}{P^£} = e = \frac{4}{3}\]

1 pound sterling = $\frac{4}{3}$ euro

$e = 1$
3. Inside Money: Money Market

a. \( l = 1, \ t = 1, 2, \ h = 1, 2 \)

b. \( u_h(x^1_h, x^2_h) = \log x^1_h + \log x^2_h \)

c. \( \omega_1 = (\omega^1_1, \omega^2_1) = (2, 8) \)

d. \( \omega_2 = (\omega^1_2, \omega^2_2) = (8, 2) \)

a. What is the equilibrium allocation \( x = ((x^1_1, x^2_1), (x^1_2, x^2_2)) \) when the money market is closed?

b. What is the Pareto optimal allocation \( x \)? Hint: you need not calculate, but you can do this for confirmation.

c. Show that the allocation \( x \) in part b is also the competitive equilibrium allocation when the money market is open. Hint: You might use the relationship between the money market equilibrium and the futures market equilibrium.

Solution:

a. 

\[
 x = ((2, 8), (8, 2)) = (\omega_1, \omega_2)
\]

because when \( P^m = 0 \) there is no intertemporal trade (no borrowing, no lending). Autarky.

b. There are many PO allocations. One is \( x_1 = (5, 5) = x_2 \) because it maximizes equal-weighted welfare \( u_1 + u_2 \) subject to \( \omega_1 + \omega_2 = (10, 10) \).

PO allocations are found by maximizing

\[
 \log(x) + \log(y) + \lambda \left[ \log(10 - x) + \log(10 - y) \right]
\]

subject to \( (0, 0) \leq (x, y) \leq (10, 10) \) where \( \lambda \geq 0 \) is the relative weight on Mr. 2.

Differentiating wrt \( x \) and \( y \) and setting to zero yields

\[
 \frac{1}{x} = \frac{\lambda}{10 - x}, \quad \frac{1}{y} = \frac{\lambda}{10 - y}.
\]

So we have

\[
 \frac{10 - x}{x} = \lambda = \frac{10 - y}{y}.
\]

If \( \lambda = 1 \), \( (x, y) = (5, 5) \).

If \( \lambda = 0 \), \( (x, y) = (10, 10) \).

If \( \lambda = \infty \), \( (x, y) = (0, 0) \).

\( x = y \) for all \( \lambda \).
c. (CP)

\[
\begin{align*}
\text{max } & \log x^1_h + \log x^2_h \\
\text{s.t. } & p^1 x^1_h + p^2 x^2_h = p^1 \omega^1_h + p^2 \omega^2_h \\
\text{or } & x^1_h + px^2_h = \omega^1_h + p\omega^2_h \\
\text{or } & x^1_h = (\omega^1_h + p\omega^2_h)/2 = x^2_h \\
\text{If } p = 1, \text{then } & x^1_h = 10/2 = 5 = x^2_h
\end{align*}
\]