Cornell University
Spring 2016
ECON 4905
Financial Fragility and the Macroeconomy

Answers for Practice Questions:
Prelim #1

1. (Outside) Money Taxation

- 1 commodity, $l = 1$, chocolate measured in ounces (oz.)
- 2 individuals, $h = 1, 2$
- taxes $\tau = (\tau_1, \tau_2)$ measured in dollars ($)
- consumption $x = (x_1, x_2)$ measured in ounces
- endowments $\omega = (\omega_1, \omega_2) = (100, 25)$ measured in ounces
- $x_h = \omega_h - P^m \tau_h$

a. What are the units in which P^m is measured?

((b) - (e)): In what follows, when is τ balanced or not, and when is τ bonafide or not? Solve for $\mathcal{P}^m$, the set of equilibrium P^m.

b. $\tau = (5, -5)$

c. $\tau = (1, -7)$

d. $\tau = (1, -1)$

e. $\tau = (-1, -1)$

f. In (b) - (e), in which cases are there multiple equilibria? What are the lessons from this for macroeconomics?

Solution:

a. ounces per dollar, oz./$.

b. balanced. bonafide.
$x_1 = 100 - 5P^m > 0, \qquad P^m < 20$
$\mathcal{P}^m = [0, 20)$.

c. not balanced. not bonafide.
$\mathcal{P}^m = \{0\}$.

d. balanced. bonafide.

$x_1 = 100 - P^m > 0, \qquad P^m < 100$

$\mathcal{P}^m = [0, 100).$

e. not balanced. not bonafide.

$\mathcal{P}^m = \{0\}.$

f. The price level is indeterminate in each of the balanced/bonafide cases: (b), (d), and (e). This emphasizes — particularly in money-finance economies — that the allocation of resources depends *as much* on beliefs as opposed to being determined solely by fundamentals (preferences and endowments).

2. Outside Money: 2 Currency Taxation

Same set-up as in (1.), but now 2 currencies: euro (€) and pound sterling (£). In each of the following solve for the exchange rate e. Give the units of e.

a. $\tau^{€} = (-1,-1)$, $\tau^{£} = (1,1)$

b. $\tau^{€} = (1,-1)$, $\tau^{£} = (-5,5)$

c. $\tau^{€} = (2,1)$, $\tau^{£} = (1,-5)$

Solution:

a.

$$\sum_h \tau_h^{€} = -2€, \quad \sum_h \tau_h^{£} = 2£$$
$$-2P^{m€} + 2P^{m£} = 0$$
$$P^{m€} : \frac{€}{\text{choc}}, \quad P^{m£} : \frac{£}{\text{choc}}, \quad \text{units}$$
$$\frac{P^{m€}}{P^{m£}} = 1 = \frac{P^{m£}}{P^{m€}}, \quad \text{exchange rates}$$
$$\frac{\frac{€}{\text{choc}}}{\frac{£}{\text{choc}}} = \frac{€}{£}, \qquad \frac{\frac{£}{\text{choc}}}{\frac{€}{\text{choc}}} = \frac{£}{€}$$
$$1 \text{ euro} = 1 \text{ pound sterling}$$
$$e = 1$$

b.

$$\sum_h \tau_h^{€} = 0, \quad \sum_h \tau_h^{£} = 0$$
$$0P^{m€} + 0P^{m£} = 0$$
$$\text{e is indeterminate}$$

c.

$$\sum_h \tau_h^{€} = 3€, \quad \sum_h \tau_h^{£} = -4£$$
$$3P^{m€} - 4P^{m£} = 0$$
$$3P^{m€} = 4P^{m£}$$
$$\frac{P^{m€}}{P^{m£}} = \frac{4}{3}$$
$$\frac{\frac{P^{€}}{\text{choc}}}{\frac{P^{£}}{\text{choc}}} = e = \frac{4}{3}$$
$$1 \text{ pound sterling} = \frac{4}{3} \text{ euro}$$
$$e = 1$$

3. Inside Money: Money Market

a. $l = 1, \quad t = 1, 2, \quad h = 1, 2$

b. $u_h(x_h^1, x_h^2) = \log x_h^1 + \log x_h^2$

c. $\omega_1 = (\omega_1^1, \omega_1^2) = (2, 8)$

d. $\omega_2 = (\omega_2^1, \omega_2^2) = (8, 2)$

a. What is the equilibrium allocation $x = ((x_1^1, x_1^2), (x_2^1, x_2^2))$ when the money market is closed?

b. What is the Pareto optimal allocation x? Hint: you need not calculate, but you can do this for confirmation.

c. Show that the allocation x in part b is also the competitive equilibrium allocation when the money market is open. Hint: You might use the relationship between the money market equilibrium and the futures market equilibrium.

Solution:

a.

$$x = ((2, 8), (8, 2)) = (\omega_1, \omega_2)$$

because when $P^m = 0$ there is no intertemporal trade (no borrowing, no lending). Autarky.

b. There are many PO allocations. One is $x_1 = (5, 5) = x_2$ because it maximizes equal-weighted welfare $u_1 + u_2$ subject to $\omega_1 + \omega_2 = (10, 10)$.

PO allocations are found by maximizing

$$\log(x) + \log(y) + \lambda\big[\log(10 - x) + \log(10 - y)\big]$$

subject to $(0, 0) \leq (x, y) \leq (10, 10)$ where $\lambda \geq 0$ is the relative weight on Mr. 2.

Differentiating wrt x and y and setting to zero yields

$$\frac{1}{x} = \frac{\lambda}{10 - x}, \quad \frac{1}{y} = \frac{\lambda}{10 - y}.$$

So we have

$$\frac{10 - x}{x} = \lambda = \frac{10 - y}{y}.$$

If $\lambda = 1, \ (x, y) = (5, 5)$.

If $\lambda = 0, \ (x, y) = (10, 10)$.

If $\lambda = \infty, \ (x, y) = (0, 0)$.

$x = y$ for all λ.

c. (CP)

$$\begin{aligned} & \max \log x_h^1 + \log x_h^2 \\ s.t. \quad & p^1 x_h^1 + p^2 x_h^2 = p^1 \omega_h^1 + p^2 \omega_h^2 \\ \text{or} \quad & x_h^1 + p x_h^2 = \omega_h^1 + p \omega_h^2 \\ & x_h^1 = (\omega_h^1 + p\omega_h^2)/2 = x_h^2 \end{aligned}$$

If $p = 1$,then

$$x_h^1 = 10/2 = 5 = x_h^2$$