The probability of being impatient is $\lambda$ and the probability being patient is $(1 - \lambda)$. The type (patient or impatient) is realized in period 1 and it is private information. The utility function is:

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma},$$

where $\gamma > 0$. Each individual has one unit of endowment in period 0. There is costless storage. If the endowment is invested in period 0 and is harvested in period 1, the rate of return is 0. If harvested late, the rate of return is $(R - 1)$. Assume that the banking industry is free-entry.

(a) What is the depositor’s *ex-ante* expected utility $W$ as a function of $c_1$ consumption in period 1, and $c_2$, consumption in period 2?

(b) Show that she prefers consumption smoothing. [That is, she prefers $(c_1 + c_2)/2$ in each state if $c_1 \neq c_2$.]

(c) What is the bank’s resource constraint $RC$? Write this down precisely. Explain $RC$ in words.

(d) What is the incentive problem? Write down the incentive constraint $IC$ precisely, and explain it in words.

(e) Solve for the optimal deposit contract for the post-deposit bank assuming that there is no run. (That is: Write down the optimal first-period payment $d_1^*$ as a function of $\lambda, R$ and $\gamma$)

(f) Show that optimal first-period payment $d_1^*$ is an increasing function of $\gamma$.

(g) Show that if $\gamma > 1$, there is a run equilibrium.

(h) What do we make of the 2 equilibria? Include in your answer a concise essay on the inadequacy of analyses limited to the post-deposit game.
2. Diamond-Dybvig Bank #2

The probability $\lambda$ of being impatient is 50%. The utility function is:

$$u(c) = 10 - \frac{1}{(0.5)\sqrt{c}}.$$ 

The rate of return to the asset harvested late is 400%, i.e.,

$$R = 5.$$

(a) What is the depositor’s ex-ante expected utility $W$ as a function of $c_1$, consumption in period 1, and $c_2$, consumption in period 2?

(b) Show that she prefers consumption smoothing.

(c) Why can’t she insure on the market or self-insure against liquidity shocks?

Assume that her endowment is 100 and that she deposits her entire endowment in the bank.

(d) What is her utility $W$ in autarky?

(e) What is her utility $W$ under perfect smoothing, i.e. when $c_1 = c_2$?

(f) What is the bank’s resource constraint $RC$? Write this down precisely. Explain this in words.

(g) What is the incentive problem? Write this down precisely and explain in words the incentive constraint $IC$.

(h) Find the optimal deposit contract for this bank. What is $W$ if there is no run?

(i) Why is there a run equilibrium for this bank?

(j) Calculate the following numerical values of ex-ante utility $W$ and and rank them in numerical ascending order: $W_{autarky}$, $W_{perfect\ smoothing}$, $W_{no\ run}$, $W_{run}$.

(k) Assume that the run probability $s$ is 0.1%. Will individuals deposit in this bank? That is, will they accept this banking contract? Explain.
3. The 2-Consumer, Pre-Deposit Bank:

The utility function of the impatient agent is

\[ u(x) = \frac{Ax^{1-\gamma}}{1 - \gamma} \]

and the utility function of the patient agent is

\[ v(x) = \frac{x^{1-\gamma}}{1 - \gamma}, \]

where \( \gamma > 1 \). The probability \( \lambda \) of being impatient is 50%. The parameter \( \gamma \) is 1.01. The endowment is \( y = 3 \). The rate of return on the asset if harvested late is 50%, i.e., \( R = 1.5 \). The probability of being first in line if 2 agents withdraw early is 50%.

(a) Solve for the numerical values of \( c^{early} \) and \( c^{wait} \). Show that they are independent of the impulse parameter \( A \).

(b) Write down the expression for the \textit{ex-ante} expected utility of the depositor, \( W \).

(c) Solve for \( \hat{c} \), the value of \( c \) that maximizes \( W \) in the post-deposit game, as a function of \( A \).

(d) Calculate the critical values \( A^{early} \) and \( A^{wait} \).

(e) Let \( A = 7 \). Describe the optimal contract for the pre-deposit game, \( c^*(s) \), as a function of \( s \), the exogenous run probability.