Clarification of Wednesday, April 6 Lecture

\[ V_h = \pi(\alpha) u_h(x_h(\alpha)) + \pi(\beta) u_h(x_h(\beta)), \]

positive marginal utility, \( u'_h > 0 \),

declining marginal utility, \( u''_h < 0 \),

so \( u_h \) is strictly increasing and strictly concave

\( h \) is risk-averse.

Assume that \( x_h(\alpha) \neq x_h(\beta) \). To show that \( h \) prefers consumption smoothing. In particular, she prefers \( \bar{x}_h = \pi(\alpha)x_h(\alpha) + \pi(\beta)x_h(\beta) \) in each state to \( (x_h(\alpha), x_h(\beta)) \).

\[
\begin{align*}
V_h(\bar{x}_h, \bar{x}_h) &= \pi(\alpha) u_h(\bar{x}_h) + \pi(\beta) u_h(\bar{x}_h) \\
&= u_h(\bar{x}_h) \quad \text{because } \pi(\alpha) + \pi(\beta) = 1 \\
&= u_h(\pi(\alpha)x_h(\alpha) + \pi(\beta)x_h(\beta)) \\
&> \pi(\alpha) u_h(x_h(\alpha)) + \pi(\beta) u_h(x_h(\beta))
\end{align*}
\]

because the concave function \( u_h \) is above its chords.

Remark:

\[
\sum_h \bar{x}_h = \sum_h \bar{x}_h(s) \quad \text{for } s = \alpha, \beta.
\]