1 Online Appendix to "Bank Runs: The Pre-Deposit game" by Shell and Zhang

In section 4.1, we analyzed the effects of the impulse demand parameter A on $\widehat{c}$, the contract supporting the *unconstrained efficient allocation*. In this appendix, we analyze the effects of the other parameters, namely p, R and b, on $\widehat{c}$. We limit our discussion to the set of parameters permiting strategic complementarity, i.e., b and R satisfying inequality (5).

1.1 Probability of impatience p

From equation (7), it is easy to show that $\widehat{c}$ is increasing in p if $AR^{b-1} < 1$. $\widehat{c}$ is equal to y if $AR^{b-1} = 1$, and $\widehat{c}$ is decreasing in p if $AR^{b-1} > 1$. Hence how p affects $\widehat{c}$ depends solely on the values of A and R. The intuition is the following: Because there is aggregate uncertainty, the economy may have 2 impatient consumers, 1 impatient consumer and 1 patient consumer, or 2 patient consumers. p changes the likelihood ratio between the first and the second scenarios.[1] The first scenario requires no cross-subsidy between the consumers. The second scenario requires a cross-subsidy, but how it is conducted depends on A and R. If $AR^{b-1} < 1$, the subsidy is from the impatient to the patient (i.e., $\widehat{c} < y$). While if $AR^{b-1} > 1$, the subsidy is from the patient to the impatient (i.e., $\widehat{c} > y$). When p increases, the second scenario becomes less likely compared to the first one and less subsidy is required (i.e., $\widehat{c}$ should be closer to y). Hence if $AR^{b-1} < 1$, $\widehat{c}$ increases as p increases. And if $AR^{b-1} > 1$, the opposite is true.

To see how different values of p correspond to the three cases of the optimal contract, note that c^{early} doesn't depend on p and

$$\lim_{p \to 1} \widehat{c} = y < c^{early}.$$

[1] The last scenario doesn't matter since the welfare in that scenario is not affected by c.

Hence we are in Case 1 whenever p is sufficiently large. Furthermore,

$$\lim_{p\to 0}\widehat{c} = \frac{2y}{(1/AR^{b-1})^{1/b}+1}$$

Hence if $\frac{2y}{(1/AR^{b-1})^{1/b}+1} \leq c^{early}$, only Case 1 obtains.

If

$$\frac{2y}{(1/AR^{b-1})^{1/b}+1} > c^{early},$$

which implies $AR^{b-1} > 1$, there is a unique level of p, denoted by p^{early}, such that

$$\widehat{c}(p^{early}) = c^{early}.$$

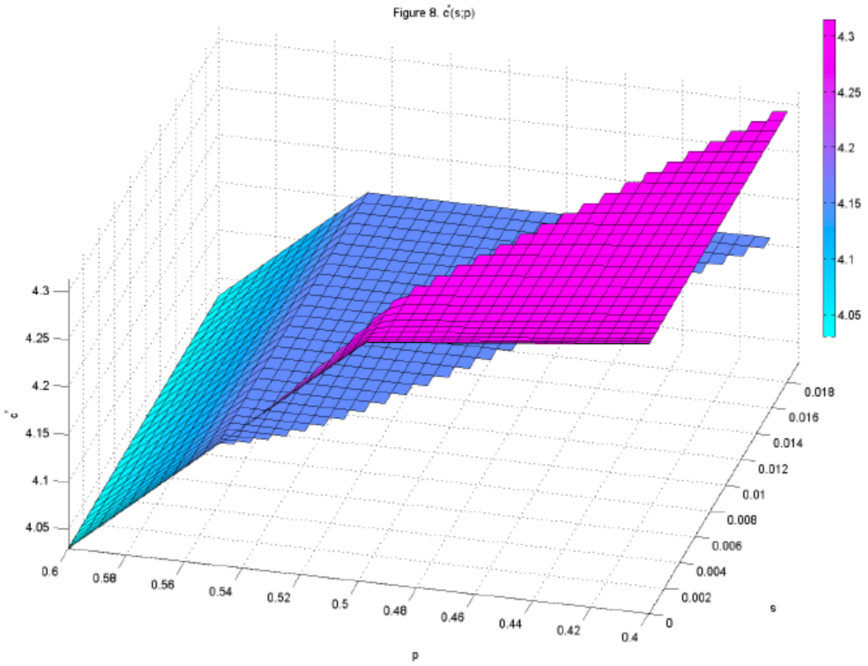
If $p \geq p^{early}$, we are in Case 1. If $p < p^{early}$, we are in Case 2 or Case 3 depending on whether $\widehat{c}(p)$ is smaller than than c^{wait} or not. Note that c^{wait} does change with p.[2]

Example A 1 *Let*

$$b = 1.01, A = 10, y = 3, R = 1.5.$$

We have $c^{early} = 4.155955$. *It is easy to check that if* $p \geq 0.548823$, *we are in Case 1. If* $0.497423 \leq p < 0.548823$, *we are in Case 2. If* $p < 0.497423$, *we are in Case 3.*

[2] In fact, c^{wait} is decreasing in p. This is quite intuitive. When a patient depositor makes his withdrawal decision, larger p implies that he expects that it is more likely that the other depositor is impatient and, therefore, withdraws early. Hence it is more costly to make the patient depositor withdraw late.



We plot c^* versus s and p in Figure 8. If $p \geq 0.548823$, we are in Case 1 and the optimal contract doesn't tolerate runs and it is equal to $\widehat{c}$ the value of which depends on p. If $0.497423 \leq p < 0.548823$, we are in Case 2 and the optimal contract is strictly decreasing in s until it levels off at the best run-proof contract $c^{early} = 4.155955$. If $p < 0.497423$, we are in Case 3 and the ICC binds when s is small.

1.2 Return on bank investment R

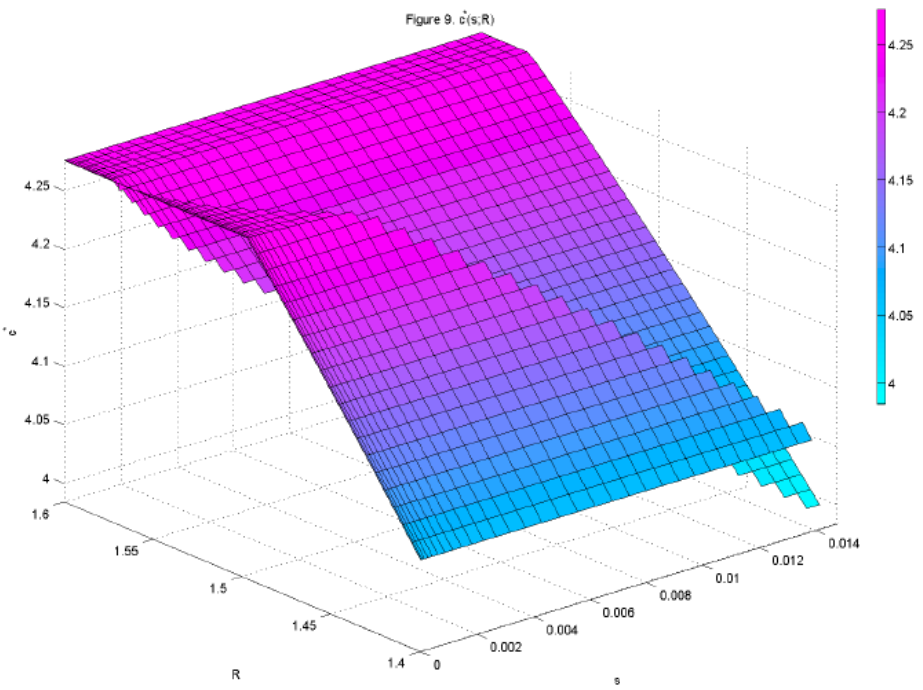
From equation (7), it is easy to show that $\widehat{c}$ is increasing in R. R affects $\widehat{c}$ by changing the optimal allocation when the economy has one impatient depositor and one patient depositor. For larger R, on the one hand, the marginal rate of transformation between the first period consumption and the second period consumption is increasing in R. On the other hand, the marginal rate of substitution between the first period consumption by the impatient depositor for the second period consumption by the patient depositor is also increasing in R. Since $b > 1$, the second effect is stronger and, therefore, the optimal allocation allows more first-period withdrawal, i.e., $\widehat{c}$ increases as R

increases. It is easy to see that both c^{early} and c^{wait} increase in R. If $\widehat{c} \leq c^{early}$, we are in Case 1. If $c^{early} < \widehat{c} \leq c^{wait}$, we are in Case 2. If $\widehat{c} > c^{wait}$, we are in Case 3.

Example A 2 *Let*

$$b = 1.01, A = 10, y = 3, p = 0.5.$$

It is easy to check that if $R \geq 1.572948$*, we are in Case 1. If* $1.497374 \leq R < 1.572948$*, we are in Case 2. If* $R < 1.497374$*, we are in Case 3.*



We plot c^* versus s and R in Figure 9. If $R \geq 1.572948$, we are in Case 1 and the optimal contract doesn't tolerate runs and it is equal to $\widehat{c}$ the value of which depends on R. If $1.497374 \leq R < 1.572948$, we are in Case 2 and the optimal contract is strictly decreasing in s until it levels off at the best run-proof contract c^{early}. Note that c^{early} increases in R. If $R < 1.497374$, we are in Case 3 and the ICC binds when s is small.

1.3 Risk aversion parameter b

The sign of $\frac{\partial \widehat{c}}{\partial b}$ is the same as the sign of

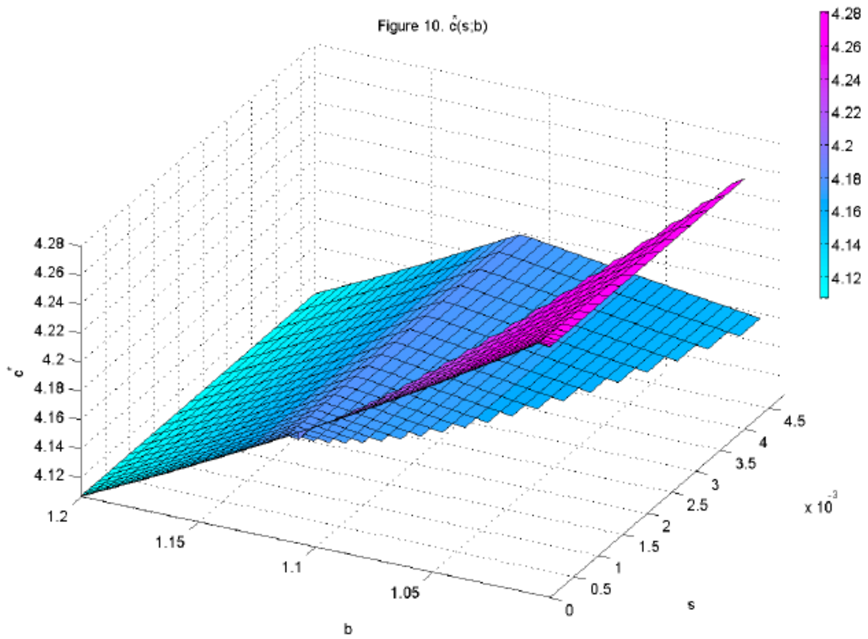
$$\ln(\frac{p}{2-p} + \frac{2(1-p)}{(2-p)AR^{b-1}}) + \frac{2(1-p)b\ln(R)}{2(1-p)+pAR^{b-1}}.$$

Hence if A is smaller than a threshold level, we have $\frac{\partial \widehat{c}}{\partial b} > 0$. Otherwise, we have $\frac{\partial \widehat{c}}{\partial b} < 0$. The intuition is the following: As b increases, consumption smoothing across the two depositors is more desirable. When A is small, $\widehat{c}$ is small and more consumption smoothing implies larger $\widehat{c}$. When A is large, $\widehat{c}$ is large and more consumption smoothing implies smaller $\widehat{c}$.

Example A 3 *Let*

$$A = 10, y = 3, p = 0.5, R = 1.5.$$

It is easy to check that if $b \geq 1.112528$*, we are in Case 1. If* $1.00524 \leq b < 1.112528$*, we are in Case 2. If* $b < 1.00524$*, we are in Case 3.*



We plot c^* versus s and b in Figure 10. If $b \geq 1.112528$, we are in Case 1 and the optimal contract doesn't tolerate runs and it is equal to $\widehat{c}$ the value of which depends on b. If $1.00524 \leq b < 1.112528$, we are in Case 2 and the optimal contract is strictly decreasing in s until it levels off at the best run-proof contract c^{early}. If $b < 1.00524$, we are in Case 3 and the ICC binds when s is small.