Economics 4905 Spring 2016
Cornell University
Financial Fragility and the Macroeconomy

Problem Set 2 Solutions
Revised Friday, April 29, 2016

1. Diamond-Dybvig Bank #1

The probability of being impatient is λ and the probability being patient is $(1-\lambda)$. The type (patient or impatient) is realized in period 1 and it is private information. The utility function is:

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma},$$

where $\gamma > 0$. Each individual has one unit of endowment in period 0. There is costless storage. If the endowment is invested in period 0 and is harvested in period 1, the rate of return is 0. If harvested late, the rate of return is $(R-1)$. Assume that the banking industry is free-entry.

(a) What is the depositor's *ex-ante* expected utility W as a function of c_1 consumption in period 1, and c_2, consumption in period 2?

Solution:
$W = \lambda u(c_1) + (1-\lambda)u(c_2)$
$W = \frac{\lambda c_1^{1-\gamma}}{1-\gamma} + \frac{(1-\lambda)c_2^{1-\gamma}}{1-\gamma}$

(b) Show that she prefers consumption smoothing.

Solution:
$u'(c) = \frac{(1-\gamma)c_1^{-\gamma}}{1-\gamma} = c^{-\gamma} > 0$
$u''(c) = -\gamma c^{-\gamma-1} < 0$

So $u(c)$ is strictly concave (And the consumer is risk-averse. Concave functions lie above their chords (Jensen's inequality):
$u(\lambda c_1 + (1-\lambda)c_2) > \lambda u(c_1) + (1-\lambda)u(c_2)$ when $c1 \neq c2$,

So $W(\bar{c}, \bar{c}) > W(c_1, c_2)$ where $\bar{c} = \lambda c_1 + (1-\lambda)c_2$.

(c) What is the bank's resource constraint RC? Write this down precisely. Explain RC in words.

Solution:

$(1-\lambda)d_2 \leq (1-\lambda d_1)R$
Where d_t is the withdrawal allowed in period $t = 1, 2$.

The LHS of the inequality is the funds to be withdrawn in period 2. The RHS is the resources available at the bank in period 2. If the inequality is violated, the bank is insolvent.

(d) What is the incentive problem? Write down the incentive constraint IC precisely, and explain it in words.

Solution:
$d_1 \leq d_2$ is the ICC.

If the inequality does not hold, every depositor will attempt to withdraw early: the depositors would not self-select correctly.

(e) Solve for the optimal deposit contract for the post-deposit bank assuming that there is no run. (That is: Write down the optimal first-period payment d_1^* as a function of λ, R and γ)

Solution:
$\max_{d_1,d_2} \lambda u(d_1) + (1-\lambda u(d_2)$
subject to RC and ICC.

The solution is

$d_1^* = \frac{1}{\lambda+(1-\lambda)R^{1/\gamma-1}}$

$d_2^* = \frac{R^{1/\gamma}}{\lambda+(1-\lambda)R^{1/\gamma-1}}$

(f) Show that optimal first-period payment d_1^* is an increasing function of γ. **Solution:**
$\frac{dd_1^*}{d\gamma} = \frac{(1-\lambda)(R^{(1/\gamma)-1}\ln(R)}{[\lambda+(1-\lambda)R^{1/\gamma}]\gamma^2} > 0$

As γ increases, risk aversion increases. Hence the optimal contract provides greater consumption-smoothing.

(g) Show that if $\gamma > 1$, there is a run equilibrium.

Solution:
If $\gamma > 1$, we have $d_1^* > 1$.
The contract is not dominant strategy incentive compatible. If other patient depositors are assumed to run, any patient depositor will run.

(h) What do we make of the 2 equilibria? Include in your answer a concise essay on the inadequacy of analyses limited to the post-deposit game.

Solution:
There are 2 equilibria to the post-deposit game. The run equilibrium alerts us to financial fragility, but the story is not complete. No one deposits in the bank if the probability of a run is 100%. We must analyze the pre-deposit game. It can be show that in this case the optimal pre-deposit contract tolerates small-probability runs.

2. Diamond-Dybvig Bank #2

The probability λ of being impatient is 50%. The utility function is:

$$u(c) = 10 - \frac{1}{(0.5)\sqrt{c}}.$$

The rate of return to the asset harvested late is 400%, i.e.,

$$R = 5.$$

(a) What is the depositor's *ex-ante* expected utility W as a function of c_1, consumption in period 1, and c_2, consumption in period 2?

Solution:
$W = \lambda u(c_1) + (1 - \lambda)u(c_2)$
$W = \lambda(10 - \frac{1}{0.5\sqrt{c_1}}) + (1 - \lambda)(10 - \frac{1}{0.5\sqrt{c_1}})$

(b) Show that she prefers consumption smoothing.

Solution:

$u''(c) < 0$. Hence, $u(c)$ is a strictly concave A concave functions lies above their chords (Jensen's inequality):
$u(\lambda c_1 + (1 - \lambda)c_2) > \lambda u(c_1) + (1 - \lambda)u(c_2)$ when $c1 \neq c2$,

That is she prefers $(\bar{c}, \bar{c})$ to (c_1, c_2) where $\bar{c} = \lambda c_1 + (1 - \lambda)c_2$.

(c) Why can't she insure on the market or self-insure against liquidity shocks?

Solution:

1. Her type is purely her own private information. The insurance company would not trust her to report her type truthfully. She would say that she is impatient even if she is not.

2. She must choose the proportion of liquid assets she holds before she knows her type. The timing does not allow for self-insurance.

Assume that her endowment is 100 and that she deposits her entire endowment in the bank.

(d) What is her utility W in autarky?
Solution:

$W_{autarky} = \lambda u(1) + (1-\lambda)u(5) = 9.85$

(e) What is her utility W under perfect smoothing, i.e. when $c_1 = c_2$?
Solution:

$W_{perfect-smoothing} = u(\lambda + (1-\lambda)5) = 9.88$

(f) What is the bank's resource constraint RC? Write this down precisely. Explain this in words.
Solution:

$(0.5)(d_2) \leq (1 - (0.5)d_1)R$

Period-2 withdrawals cannot exceed period-2 bank resources.

(g) What is the incentive problem? Write this down precisely and explain in words the incentive constraint IC.
Solution:

$d_1 \leq d_2$ (ICC)

If ICC does not hold, everyone will seek to withdraw in period 1.

(h) Find the optimal deposit contract for this bank. What is W if there is no run?
Solution:
$\max_{d_1,d_2} \lambda u(d_1) + (1 - \lambda u(d_2)$

subject to RC and ICC.

$d_1^* = 1.2620, d_2^* = 3.6901$
$W_{non-run} = 9.86$

(i) Why is there a run equilibrium for this bank?
Solution:

$d_1^* = 1.2620 > 1$

(j) Calculate the following numerical values of *ex-ante* utility W and and rank them in numerical ascending order: $W_{autarky}$, $W_{perfect\ smoothing}$, $W_{no\ run}$, W_{run}.
Solution:
$W_{run} = (1/d_1^*)u(d_1^*) = 6.5132$

$W_{run} < W_{autarky} < W_{non-run} < W_{perfectsmoothing}$

(k) Assume that the run probability s is 0.1%. Will individuals deposit in this bank? That is, will they accept this banking contract? Explain.
Solution:

$(.001)W_{run} + (.999)W_{nonrun} = 9.86 > W_{autarky}$

Consumers will deposit in the bank even if there is a .1% run probability.

3. The 2-Consumer, Pre-Deposit Bank:

The utility function of the impatient agent is

$$u(x) = \frac{Ax^{1-\gamma}}{1-\gamma}$$

and the utility function of the patient agent is

$$v(x) = \frac{x^{1-\gamma}}{1-\gamma},$$

where $\gamma > 1$. The probability λ of being impatient is 50%. The parameter γ is 1.01. The endowment is y = 3. The rate of return on the asset if harvested late is 50%, i.e., R = 1.5. The probability of being first in line if 2 agents withdraw early is 50%.

(a) Solve for the numerical values of c^{early} and c^{wait}. Show that they are independent of the impulse parameter A.

Solution:

Refer to the lecture slides on the pre-deposit game and the related paper. From slide 17, we have:

$c^{wait} = 4.280878 > 4.155955 = c^{early}$ Hence, the parameters are "usual".

(b) Write down the expression for the *ex-ante* expected utility of the depositor, W.

Solution:

$c^*(s) = \arg \max_{c \in [0, c^{wait}]} W(c, s)$

where

$$W(c; s) = \begin{cases} \hat{W}(c) & c \leq c^{early} \\ (1-s)\hat{W}(c) + sW^{run}(c) & c \in (c^{early}, c^{wait}) \end{cases}$$

Where $\hat{W}(c)$ is welfare in the post-deposit game non-run equilibrium $W^{run}(c)$ is the welfare in the run equilibrium.

(c) Solve for $\hat{c}$, the value of c that maximizes W in the post-deposit game, as a function of A.

Solution:

$\hat{c} = \frac{2y}{(\frac{p}{1-p} + \frac{2(1-p)}{(2-p)AR^{\gamma-1}})^{1/\gamma}}$

$\frac{6}{1/3+(\frac{2}{3}(A1.5)^{1.01})^{1/1.01}+1}$

(d) Calculate the critical values A^{early} and A^{wait}.
Solution:

$A^{early} = 6.217686 < 10.27799 = A^{wait}$

(e) Let A = 7. Describe the optimal contract for the pre-deposit game, $c^*(s)$, as a function of s, the exogenous run probability.
Solution:

$\hat{c}(A = 7) = 4.19054$.

We are in Case 2. $c^*(s = 0) = \hat{c} = 4.19054$. c is strictly declining in s until $s = s_0 = .0003468$. For $s > s_0$, c is independent of s and equal to $c^{early} = 4.155955$.