Economics 4905 Spring 2016
Cornell University
Financial Fragility and the Macroeconomy

Problem Set 2
Due: Wednesday, April 20, 2016

1. Diamond-Dybvig Bank #1

The probability of being impatient is λ and the probability being patient is $(1-\lambda)$. The type (patient or impatient) is realized in period 1 and it is private information. The utility function is:

$$u(c) = \tfrac{c^{1-\gamma}}{1-\gamma},$$

where $\gamma > 0$. Each individual has one unit of endowment in period 0. There is costless storage. If the endowment is invested in period 0 and is harvested in period 1, the rate of return is 0. If harvested late, the rate of return is $(R-1)$. Assume that the banking industry is free-entry.

(a) What is the depositor's *ex-ante* expected utility W as a function of c_1 consumption in period 1, and c_2, consumption in period 2?

(b) Show that she prefers consumption smoothing. [That is, she prefers $(c_1+c_2)/2$ in each state if $c_1 \neq c_2$.]

(c) What is the bank's resource constraint RC? Write this down precisely. Explain RC in words.

(d) What is the incentive problem? Write down the incentive constraint IC precisely, and explain it in words.

(e) Solve for the optimal deposit contract for the post-deposit bank assuming that there is no run. (That is: Write down the optimal first-period payment d_1^* as a function of λ, R and γ)

(f) Show that optimal first-period payment d_1^* is an increasing function of γ.

(g) Show that if $\gamma > 1$, there is a run equilibrium.

(h) What do we make of the 2 equilibria? Include in your answer a concise essay on the inadequacy of analyses limited to the post-deposit game.

2. Diamond-Dybvig Bank #2

The probability λ of being impatient is 50%. The utility function is:

$$u(c) = 10 - \frac{1}{(0.5)\sqrt{c}}.$$

The rate of return to the asset harvested late is 400%, i.e.,

$$R = 5.$$

(a) What is the depositor's *ex-ante* expected utility W as a function of c_1, consumption in period 1, and c_2, consumption in period 2?

(b) Show that she prefers consumption smoothing.

(c) Why can't she insure on the market or self-insure against liquidity shocks?

Assume that her endowment is 100 and that she deposits her entire endowment in the bank.

(d) What is her utility W in autarky?

(e) What is her utility W under perfect smoothing, i.e. when $c_1 = c_2$?

(f) What is the bank's resource constraint RC? Write this down precisely. Explain this in words.

(g) What is the incentive problem? Write this down precisely and explain in words the incentive constraint IC.

(h) Find the optimal deposit contract for this bank. What is W if there is no run?

(i) Why is there a run equilibrium for this bank?

(j) Calculate the following numerical values of *ex-ante* utility W and and rank them in numerical ascending order: $W_{autarky}$, $W_{perfect\ smoothing}$, $W_{no\ run}$, W_{run}.

(k) Assume that the run probability s is 0.1%. Will individuals deposit in this bank? That is, will they accept this banking contract? Explain.

3. The 2-Consumer, Pre-Deposit Bank:

The utility function of the impatient agent is

$$u(x) = \frac{Ax^{1-\gamma}}{1-\gamma}$$

and the utility function of the patient agent is

$$v(x) = \frac{x^{1-\gamma}}{1-\gamma},$$

where $\gamma > 1$. The probability λ of being impatient is 50%. The parameter γ is 1.01. The endowment is y = 3. The rate of return on the asset if harvested late is 50%, i.e., R = 1.5. The probability of being first in line if 2 agents withdraw early is 50%.

(a) Solve for the numerical values of c^{early} and c^{wait}. Show that they are independent of the impulse parameter A.

(b) Write down the expression for the *ex-ante* expected utility of the depositor, W.

(c) Solve for $\hat{c}$, the value of c that maximizes W in the post-deposit game, as a function of A.

(d) Calculate the critical values A^{early} and A^{wait}.

(e) Let A = 7. Describe the optimal contract for the pre-deposit game, $c^*(s)$, as a function of s, the exogenous run probability.