ECON 4905 FALL 2016

• Last Name:
• First Name:
  • Nickname (if any)
• Best way to reach you:
• Class:
• Department:
• Economics Background:
• Math Background:
• Other:
• Relevant Employment:
• Goals:
Outside Money

• Demand by agent $h$

\[ x_h = \{ x^{1}_h, \ldots, x^{i}_h, \ldots, x^{\ell}_h \} > 0, \ x_h \in \mathbb{R}^{\ell}_{++} \]

• Endowments

\[ \omega_h = \{ \omega^{1}_h, \ldots, \omega^{i}_h, \ldots, \omega^{\ell}_h \} > 0, \ \omega_h \in \mathbb{R}^{\ell}_{++} \]

• Agents range from $h = 1, \ldots, n$
Outside Money, Continued: Taxes

- Taxes, in dollars
  \[ \mathcal{T} = \{\mathcal{T}_1, \mathcal{T}_2, \ldots, \mathcal{T}_h, \ldots, \mathcal{T}_n\}, \quad \mathcal{T}_h \in \mathbb{R}^n \]

- If \( \mathcal{T}_h > 0 \), then Mr. \( h \) is taxed

- If \( \mathcal{T}_h < 0 \), then Mr. \( h \) is subsidized

- If \( \mathcal{T}_h = 0 \), then \( h \) is neither taxed nor subsidized.
Utility

• Utility: $u_h(x_h)$

• Assume, for simplicity, that $u'_h(x_h) > 0$ and $u''_h(x_h) < 0$. 
Prices

• Prices

\[ p = \{ p^1, \ldots, p^i, \ldots, p^\ell \} > 0, \quad p \in \mathbb{R}^\ell_+ \]

• Choose *numeraire*, say \( p^1 = 1 \).

• Price of money in terms of commodity 1: \( p^m \geq 0 \)

\[ P^m = \frac{p^m}{p^1} = p^m \geq 0 \]

• Commodity (ex: chocolate) price of money. Notice that money can be worthless, such that \( P^m = 0 \).
Difference between money and other commodities

• The price of chocolate is always positive because it is desired and in short supply.
• Money is desired only if its price is positive
  \[ P^m \sim \frac{1}{\text{general price level}} \]
• Suggests indeterminacy of equilibrium
  • Source of some financial instability
Equilibrium

- Supply of goods = demand for goods
- Supply of money = demand for money
- Mr. h’s problem:

\[(CP) \begin{cases}
\max u_h(x_h) \\
\text{subject to} \\
p \cdot x_h = p \cdot \omega_h - P^m \tau_h, \text{ or} \\
p \cdot (x_h - \omega_h) + P^m \tau_h = 0
\end{cases}\]
Equilibrium, Continued

• Budget constraint of $h$ expanded:
  \[ \sum_{i=1}^{\ell} p^i x^i_h = \sum_{i=1}^{\ell} p^i \omega^i_h - P^m \tau_h \]

  • For $h = 1, \ldots, n$

• Let $x_h$ satisfy the CP at prices $(p, P^m)$ for $h = 1,\ldots,n$.

• Then $(p, P^m) \in \mathbb{R}^\ell_{++} \times \mathbb{R}_+$ and $x \in \mathbb{R}^{\ell \times n}_{++}$ are equilibrium values of the goods when markets clear, i.e.
  \[ \sum_{h=1}^{n} x_h = \sum_{h=1}^{n} \omega_h \]
Bonafide Taxes and Balanced Taxes

- $\mathcal{T}$ is said to be balanced if $\sum_h \tau_h = 0$

- $\mathcal{T}$ is said to be *bonafide* if it allows for some equilibrium with *some* positive price of money, such that $P^m > 0$.

- Why do we call this bonafide?
  - Hint: *bona fide*
Dollar Taxation, Continued

• Summing over budget constraints allows us to conclude that if
  \[ p \cdot x_h = p \cdot \omega_h - P^m \tau_h = 0 \]

• Then
  \[ p \cdot \sum_{h} x_h = p \cdot \sum_{h} \omega_h - P^m \sum_{h} \tau_h = 0 \]

• In equilibrium
  \[ \sum_{h=1}^{n} x_h = \sum_{h=1}^{n} \omega_h. \text{ Hence, } P^m \sum_{h=1}^{n} \tau_h = 0 \]

• Therefore, either \( P^m = 0 \), or \( \sum_{h=1}^{n} \tau_h = 0 \), or both.
Dollar Taxation, Continued Further

• Hence: an imbalanced tax (i.e. fiscal) policy (where $\sum_{h=1}^{n} \tau_h \neq 0$) is not bonafide in finite economies.

• Thus:
  • If taxes are bonafide, then they are also balanced.

• Also true, for finite economies
  • If taxes are balanced, then they are also bonafide.
  • This will be shown in the tax-adjusted simple graphical analysis:
Balanced $\Rightarrow$ Bonafide

• For simplicity, $\ell = 2$ and there are no “corners.”

• Define tax-adjusted endowment

  $\tilde{\omega} = (\omega^1_h, \omega^2_h)$ by

  $\tilde{\omega}^1_h = \omega^1_h - P^m \tau_h,$

  $\tilde{\omega}^2_h = \omega^2_h$

• Forget $\tau_h \leq 0$. Focus on $\tau_h > 0$. 
Generalization

• Does not require nice indifference curves.

• Can be extended to $\ell$ commodities

• We will extend the analysis to (finite) dynamic economies with perfect capital (borrowing-lending) markets, where balancedness requires retirement of the public debt (Ricardo)

• Does not extend to the overlapping-generations economy, allowing for rational, non-bursting bubbles.
Some Take-Aways

- The equilibrium $P^m$ is not determinate, and is thus a source of fragility. The price of a paper asset in terms of a real asset or commodity depends on the beliefs of people.

- $P^m = 0$ is always an equilibrium value.
Future Lectures

• Analysis of this lecture extends to finite, dynamic economies.
• Does not extend to overlapping-generations models with infinite horizons, allowing for bubbles.