Lecture 3, Slide 1

Worked Examples

- Outside money and the Price Level
- 2 Currencies, Exchange Rates, and the Price Levels

Outside Money and the Price Level:

- **Example 1:**
  - $\ell = 1$
  - $n = 4$
  - $\omega = (\omega_1, \omega_2, \omega_3, \omega_4) = (20, 15, 10, 5)$
  - $\tau = (\tau_1, \tau_2, \tau_3, \tau_4) = (1, 1, 1, -1)$

Outside Money and the Price Level:

- **Example 1, Continued:**
  - $\tau = (\tau_1, \tau_2, \tau_3, \tau_4) = (1, 1, 1, -1)$
  - $\sum \tau_h = 2 \neq 0$
  - Not balanced
  - $P^m = \{0\}$
  - $x = (x_1, x_2, x_3, x_4) = (20, 15, 10, 5)$
  - Autarky

Example 2

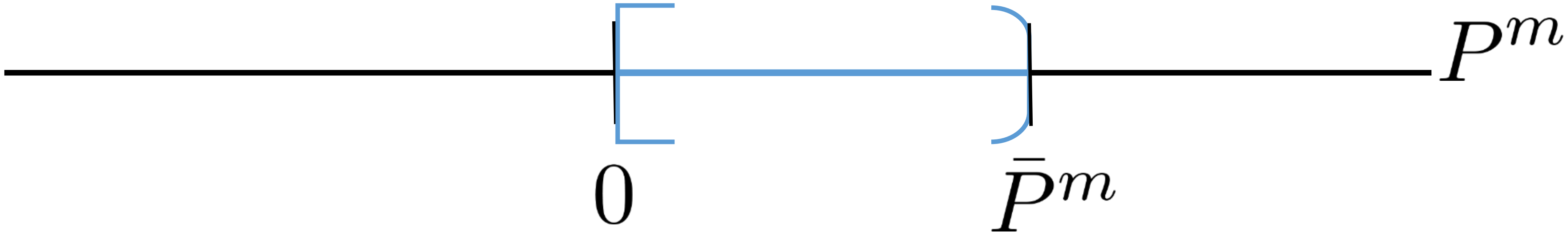
- As before, but $\tau = (5, 2, -2, -5)$
- $\sum \tau_h = 0$
- Balanced
- $x_1 = 20 - 5P^m > 0 \;\Rightarrow\; P^m < 4$
- $x_2 = 15 - 2P^m > 0 \;\Rightarrow\; P^m < \frac{15}{2}$
- $x_3 = 10 + 2P^m$
- $x_4 = 5 + 5P^m$

Therefore
- $0 \leq P^m < 4 < \frac{15}{2}$
- $P^m \in [0, 4)$

Example 2, Continued

- As $\mathcal{P}^m = [0, 4)$
- $\{(x_1, x_2, x_3, x_4) | x_1 = 20 - 5P^m,\ x_2 = 15 - 2P^m,$
  $x_3 = 10 + 2P^m,\ x_4 = 5 + 5P^m,\ P^m \in [0, 4)\}$
- $x = (x_1, x_2, x_3, x_4)$ not independent

- **In general, for $\ell = 1$, we have**



- For balanced policies, we have
  - $P^m \in [0, \bar{P}^m)$
  - $\bar{P}^m \geq 0$

Quantity Theory of Money (QTM)

Let $\tau' = 2\tau$ as from Example 2

- $\tau' = 2(5, 2, -2, -5) = (10, 4, -4, -10)$
- $x_1 = 20 - 10P^m > 0 \;\Rightarrow\; P^m < 2$
- $x_2 = 15 - 4P^m > 0 \;\Rightarrow\; P^m < \dfrac{15}{4}$
- $0 < P^m < 2 < \dfrac{15}{4}$
- $P^m \in [0, 2)$
- $\bar{P}^m = 2$

QTM Continued

$$P^m \in [0, 2),\ \mathcal{P}^m = [0, 2)$$

- This is a statement about sets, not price levels
- If everyone believes QTM, then QTM is REE
- If not, not

Take-Away

- Indeterminacy of the price level
- Beliefs about P^m and fundamentals ω jointly determine outcomes
- Beliefs matter
- The quantity theory of money is (too) subtle. Doubling τ will affect $\mathcal{P}^m$ but not necessarily according to QTM.

Lecture 3, Slide 9

Two Currencies, R and B:

- Bi-metalism in the US
- “Cross of Gold” speech
- Borrowers hurt by deflation

Two Currencies, R and B:

- $\ell = 1,\ n = 5,\ \omega = (25, 20, 15, 10, 5)$
- $\tau^B = (1, 1, 1, -1, -1), \quad \tau^R = (1, 1, -1, -1, -1)$
- $\sum \tau_h^B = 1, \quad \sum \tau_h^R = -1$
  - $P^B \sum \tau_h^B + P^R \sum \tau_h^R = 0$
  - $P^B - P^R = 0 \ \Rightarrow \quad P^B = P^R$

Two Currencies, R and B:

- $x_1 = 25 - P^B - P^R = 25 - 2P^B > 0 \quad \Rightarrow P^B < \frac{25}{2}$
- $x_2 = 20 - P^B - P^R \quad \Rightarrow P^B < 10$
- $x_3 = 15 - P^B + P^R = 15$
- $0 \leq P^B < 10 < \frac{25}{2}$

- $\mathcal{P}^m = \{P^B, P^R | P^B = P^R,\ P^B \in [0, 10)\}$
  - $$\{(x_1, x_2, x_3, x_4, x_5) | x_1 = 25 - 2P^B,\ x_2 = 20 - 2P^B, \\ x_3 = 15,\ x_4 = 10 + 2P^B\} \\ x_5 = 5 + 2P^B,\ P^B \in [0, 10)\}$$

- The elements of *x* are not independent. They are constrained by $\mathcal{P}^m$.

In General

- If $\sum \tau_h^B$ and $\sum \tau_h^R$ agree in sign, then $p^B = p^R = 0.$
- If $\sum \tau_h^B$ and $\sum \tau_h^R$ disagree in sign, then either the exchange rate is
  - $$\frac{P^B}{P^R} = -\frac{\sum \tau_h^R}{\sum \tau_h^B}$$
  - Or $P^B = P^R = 0$

- Why?
- If $\sum \tau_h^B = \sum \tau_h^R = 0$, then $\frac{P^B}{P^R}$ is indeterminate.
- Why?