Worked Examples

• Outside money and the Price Level
• 2 Currencies, Exchange Rates, and the Price Levels
Outside Money and the Price Level:

**Example 1:**

- $\ell = 1$
- $n = 4$
- $\omega = (\omega_1, \omega_2, \omega_3, \omega_4) = (20, 15, 10, 5)$
- $\tau = (\tau_1, \tau_2, \tau_3, \tau_4) = (1, 1, 1, -1)$
Outside Money and the Price Level:

• Example 1, Continued:
  
  • \( \tau = (\tau_1, \tau_2, \tau_3, \tau_4) = (1, 1, 1, -1) \)
  
  • \( \sum \tau_h = 2 \neq 0 \)
  
  • Not balanced
  
  • \( P^m = \{0\} \)
  
  • \( x = (x_1, x_2, x_3, x_4) = (20, 15, 10, 5) \)
  
  • Autarky
Example 2

• As before, but \( \tau = (5, 2, -2, -5) \)
• \( \sum \tau_h = 0 \)
• Balanced
  • \( x_1 = 20 - 5P^m > 0 \Rightarrow P^m < 4 \)
  • \( x_2 = 15 - 2P^m > 0 \Rightarrow P^m < \frac{15}{2} \)
  • \( x_3 = 10 + 2P^m \)
  • \( x_4 = 5 + 5P^m \)

Therefore
• \( 0 \leq P^m < 4 < \frac{15}{2} \)
• \( P^m \in [0, 4) \)
Example 2, Continued

- As $\mathcal{P}^m = [0, 4)$
- $\{(x_1, x_2, x_3, x_4) | x_1 = 20 - 5P^m, \ x_2 = 15 - 2P^m,$
  \[ \ x_3 = 10 + 2P^m, \ x_4 = 5 + 5P^m, \ P^m \in [0, 4) \}$

- $x = (x_1, x_2, x_3, x_4)$ not independent
• In general, for $\ell = 1$, we have

• For balanced policies, we have
  • $P^m \in [0, \bar{P}^m)$
  • $\bar{P}^m \geq 0$
Quantity Theory of Money (QTM)

Let $\tau' = 2\tau$ as from Example 2

- $\tau' = 2(5, 2, -2, -5) = (10, 4, -4, -10)$
- $x_1 = 20 - 10P^m > 0 \Rightarrow P^m < 2$
- $x_2 = 15 - 4P^m > 0 \Rightarrow P^m < \frac{15}{4}$
- $0 < P^m < 2 < \frac{15}{4}$
- $P^m \in [0, 2)$
- $\bar{P}^m = 2$
QTM Continued

\[ P^m \in [0, 2), \ P^m = [0, 2) \]

- This is a statement about sets, not price levels
- If everyone believes QTM, then QTM is REE
- If not, not
Take-Away

• Indeterminacy of the price level
• Beliefs about $P^m$ and fundamentals $\omega$ jointly determine outcomes
• Beliefs matter
• The quantity theory of money is (too) subtle. Doubling $\tau$ will affect $P^m$ but not necessarily according to QTM.
Two Currencies, R and B:

- Bi-metalism in the US
- “Cross of Gold” speech
- Borrowers hurt by deflation
Two Currencies, R and B:

- $\ell = 1$, $n = 5$, $\omega = (25, 20, 15, 10, 5)$
- $\tau^B = (1, 1, 1, -1, -1)$, $\tau^R = (1, 1, -1, -1, -1)$
- $\sum \tau^B_h = 1$, $\sum \tau^R_h = -1$
  - $P^B \sum \tau^B_h + P^R \sum \tau^R_h = 0$
  - $P^B - P^R = 0 \Rightarrow P^B = P^R$
Two Currencies, R and B:

- \[ x_1 = 25 - P^B - P^R = 25 - 2P^B > 0 \Rightarrow P^B < \frac{25}{2} \]
- \[ x_2 = 20 - P^B - P^R \Rightarrow P^B < 10 \]
- \[ x_3 = 15 - P^B + P^R = 15 \]
- \[ 0 \leq P^B < 10 < \frac{25}{2} \]
\( \mathcal{P}^m = \{ P^B, P^R | P^B = P^R, P^B \in [0, 10) \} \)

\( \{(x_1, x_2, x_3, x_4, x_5) | x_1 = 25 - 2P^B, x_2 = 20 - 2P^B, x_3 = 15, x_4 = 10 + 2P^B \} \)

\( x_5 = 5 + 2P^B, P^B \in [0, 10) \)\)

• The elements of \( x \) are not independent. They are constrained by \( \mathcal{P}^m \).
In General

- If \( \sum \tau^B_h \) and \( \sum \tau^R_h \) agree in sign, then \( p^B = p^R = 0 \).

- If \( \sum \tau^B_h \) and \( \sum \tau^R_h \) disagree in sign, then either the exchange rate is
  - \( \frac{P^B}{P^R} = - \frac{\sum \tau^R_h}{\sum \tau^B_h} \)
  - Or \( P^B = P^R = 0 \)
• Why?

• If \( \sum \tau_h^B = \sum \tau_h^R = 0 \), then \( \frac{P^B}{P_R} \) is indeterminate.

• Why?