1. Diamond-Dybvig Bank #1

The probability $\lambda$ of being impatient is 50%. The utility function is:

$$u(c) = 10 - \frac{1}{0.5\sqrt{c}}.$$ 

The rate of return to the asset harvested late is 400%, i.e.,

$$R = 5.$$

(a) What is the depositor’s *ex-ante* expected utility $W$ as a function of $c_1$, consumption in period 1, and $c_2$, consumption in period 2?

(b) Show that the depositor prefers consumption smoothing.

(c) Why can’t the depositor insure on the market or self-insure against liquidity shocks?

   Assume that her endowment is 100 and that she deposits her entire endowment in the bank.

(d) What is her utility $W$ in autarky?

(e) What is her utility $W$ under perfect smoothing, i.e. when $c_1 = c_2$?

(f) What is the bank’s resource constraint $RC$? Write this down precisely. Explain this in words.

(g) What is the incentive problem? Write this down precisely and explain in words the incentive constraint $IC$. 
(h) Find the optimal deposit contract for this bank. What is $W$ if there is no run?

(i) Why is there a run equilibrium for this bank?

(j) Calculate the following numerical values of $ex-ante$ utility $W$ and rank them in numerical ascending order: $W_{autarky}$, $W_{perfect smoothing}$, $W_{no run}$, $W_{run}$.

(k) Assume that the run probability $s$ is 2%. Will individuals deposit in this bank? That is, will they accept this banking contract? Explain.
2. Diamond-Dybvig Bank #2

The probability of being impatient is $\frac{1}{2}$. The type (patient or impatient) is realized in period 1 and it is private information. The utility function is:

$$u(c) = \frac{c^{1-\gamma}}{-\gamma},$$

where $\gamma = \frac{1}{2}$. Each individual has one unit of endowment in period 0. There is costless storage. If the endowment is invested in period 0 and is harvested in period 1, the rate of return is 0. If harvested late, the rate of return is 5. Assume that the banking industry is free-entry.

(a) What is the depositor’s \textit{ex-ante} expected utility $W$ as a function of $c_{1}$ consumption in period 1, and $c_{2}$, consumption in period 2?

(b) Show that the depositor prefers consumption smoothing. [That is, she prefers $(c_{1} + c_{2})/2$ in each state if $c_{1} \neq c_{2}$.]

(c) What is the bank’s resource constraint $RC$? Write this down precisely and explain in words.

(d) What is the incentive problem? Write down the incentive constraint $IC$ precisely, and explain it in words.

(e) Solve for the optimal deposit contract for the post-deposit bank assuming that there is no run. (That is: Write down the optimal first-period payment $d_{1}^{*}$ as a function of $\lambda, R$ and $\gamma$)

(f) Is there a run equilibrium to this “optimal contract”?
3. The 2-Consumer, Pre-Deposit Bank:

The utility function of the impatient agent is

$$u(x) = \frac{Ax^{1-\gamma}}{1-\gamma}$$

and the utility function of the patient agent is

$$v(x) = \frac{x^{1-\gamma}}{1-\gamma},$$

where $\gamma > 1$. The probability $\lambda$ of being impatient is 50%. The parameter $\gamma$ is 1.01. The endowment is $y = 3$. The rate of return on the asset if harvested late is 50%, i.e., $R = 1.5$. The probability of being first in line if 2 agents withdraw early is 50%.

(a) Solve for the numerical values of $c^{\text{early}}$ and $c^{\text{wait}}$. Show that they are independent of the impulse parameter $A$.

(b) Write down the expression for the ex-ante expected utility of the depositor, $W$.

(c) Solve for $\hat{c}$, the value of $c$ that maximizes $W$ in the post-deposit game, as a function of $A$.

(d) Calculate the critical values $A^{\text{early}}$ and $A^{\text{wait}}$.

(e) Let $A = 7$. Describe the optimal contract for the pre-deposit game, $c^*(s)$, as a function of $s$, the exogenous run probability.