Macroeconomics I, Part 2
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MW 8:40-9:55AM, 202 Uris Hall

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Focus of Part 1

- DSGE, or RBC
- Single agent, infinite life
- Rational Expectations

Focus of Part 2

- OG, or OLG
- Heterogeneous agents
- Finite lives, overlapping
- Demography
- Coordination of rational beliefs
- Money
- Finance
In Part 2, we will continue to focus on rational expectations, but with expectations not necessarily coordinated on the solution to a planning problem.
Money

- Outside Money
- Inside Money
Outside Money: Taxes denominated in money, say dollars

- The static case: see Balasko-Shell article in the McKenzie volume, or simply click on the URL in the reading list
- commodities: \( i = 1, \ldots, l \)
- agents: \( h = 1, \ldots, n \)
- consumption: \( x^i_h > 0 \)
- endowment: \( \omega^i_h > 0 \)
- commodity price: \( p^i > 0 \) with \( p^1 = 1 \)
  commodity price of money: \( p^m \geq 0 \)
- lump-sum tax: \( \tau_h \)
Equilibrium

- $x_h = (x_1^h, \ldots, x_i^h, \ldots, x_l^h) \in \mathbb{R}_{++}^l$
- $\omega_h = (\omega_1^h, \ldots, \omega_i^h, \ldots, \omega_l^h) \in \mathbb{R}_{++}^l$
- $p = (p_1^i, \ldots, p_i^i, \ldots, p_l^i) \in \mathbb{R}_{++}^l$
- $p^m \in \mathbb{R}_+$
- $\tau = (\tau_1, \ldots, \tau_h, \ldots, \tau_n) \in \mathbb{R}^n$

Consumer Problem (CP):

$$\max_{x_h > 0} u_h(x_h)$$

s.t.

$$p \cdot x_h = p \cdot \omega_h - p^m \tau_h$$
(p, p^m) \in \mathbb{R}^l_+ \times \mathbb{R}_+ \text{ is said to be an competitive equilibrium if}

\sum_{h=1}^n x_h = \sum_{h=1}^n \omega_h, \text{ where}

x_h \text{ solves CP for } h = 1, \ldots, n
The tax vector $\tau$: 

- is said to be *bonafide*, if given $\tau \in \mathbb{R}^n$ there is some equilibrium $(p, p^m)$ with $p^m > 0$
- is said to be *balanced* if we have

$$\sum_{h=1}^{n} \tau_h = 0$$
Money Taxation:

- If $\tau$ is not balanced, then $\tau$ is not bonafide
- Hence
  $\tau$ bonafide $\rightarrow$ $\tau$ balanced
- Proof

\[
p \cdot x_h = p \cdot \omega_h - p^m \tau_h
\]
\[
p \cdot \sum x_h = p \cdot \sum \omega_h - p^m \sum \tau_h
\]
\[
p^m \sum \tau_h = 0
\]

$p^m = 0$ or $\sum \tau_h = 0$ or both
Balanced $\rightarrow$ Bonafide

- True, but not so simple
- sketch of proof

Define

$$\tilde{\omega}_h = (\tilde{\omega}_h^1, \tilde{\omega}_h^i)$$

$$= (\omega_h^1 - p_m^T_h, \omega_h^i)$$

$\equiv$ tax-adjusted endowment
The set of equilibrium money prices: $\mathbb{P}^m$

For $l = 1$, $\mathbb{P}^m$ is an interval if $\tau$ is bonafide. Otherwise, $\mathbb{P}^m = \{0\}$
worked examples

1. \(n = 5, \ l = 1\)

\[\omega = (150, 80, 75, 25, 10)\]

\[\tau = (40, 15, 10, -10, -30)\]

\[\sum \tau_h = 25 \neq 0\]

\(\tau\) not balanced, \(\mathbb{P}^m = \{0\}\)

2. same \(\omega\)

\[\tau = (45, 15, 0, -10, -50)\]

\[\sum \tau_h = 0, \ \tau\) balanced\)

Mr. h’s problem is

\[\max u_h(x_h) \ \text{s.t.}\]

\[x_h + P^m \tau_h = \omega_h\]
$P^m$ is the set of all $P^m$ such that $x_h > 0$ for $h = 1, \ldots, n$:

$$P^m < \min_h \omega_h \max(0, \tau_h)$$
For Mr.1
\[ 150 - 45P^m > 0, \quad P^m < \frac{150}{45} = 3\frac{1}{3} \]
For Mr.2
\[ 80 - 15P^m > 0, \quad P^m < \frac{80}{15} = 5.33 > 3\frac{1}{3} \]
Hence
\[ P^m = [0, \bar{P}^m) = [0, 3\frac{1}{3}) \]
No calculations are needed for \( h = 3, 4, 5 \). Why?
Two Monies

- Bi metalism
- Pounds Sterling and Guineas
- Thalers and Pieces of 8
- Etc!

R$ and B$, \( l = 1, \ n = 3 \)
\( \tau^R = (2, 1, 0), \quad \tau^B = (5, 3, -12) \)

max \( u_h(x_h) \)

s.t.
\[ x_h + P^R \tau^R_h + P^B \tau^B_h = \omega_h \]

\[
\sum x_h + P^R \sum \tau^R_h + P^B \sum \tau^B_h = \sum \omega_h
\]

\[
P^R \sum \tau^R_h + P^B \sum \tau^B_h = 0
\]

\[
e_{RB} = \frac{P^R}{P^B} = -\frac{\sum \tau^B_h}{\sum \tau^R_h} = -\frac{-4}{3} = \frac{4}{3}
\]