Baumol’s transactions demand for cash

A succinct summary of Baumol’s paper and why it matters for the macro-economy

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Assumptions

• One individual
• Rational behaviour $\rightarrow$ cost minimization
• $T =$ payment in € in a steady stream
• $i =$
  - interest cost in € (borrowing money)
  - or, opportunity cost in € (withdrawal from investment)
• $b =$ “brokers’ fees”
• $C =$ amount of cash borrowed/withdrawn
For example:
The cost function

1) Cost of «Brokers’ fees»:
   
   \[ b \cdot \frac{T}{C} \]

2) Interest cost of holding cash:
   
   \[ i \cdot \frac{C}{2} \]
The cost function

1) Cost of «Brokers’ fees»:
   \[ b \cdot \frac{T}{C} \]
   - number of withdrawals

2) Interest cost of holding cash:
   \[ i \cdot \frac{C}{2} \]
   - average cash holding
The cost function

1) Cost of «Brokers’ fees»:
   \[ b \cdot \frac{T}{C} \]
   - number of withdrawals

2) Interest cost of holding cash:
   \[ i \cdot \frac{C}{2} \]
   - average cash holding

1+2) Total cost function:
   \[ TC(C) = b \frac{T}{C} + i \frac{C}{2} \]
How many withdrawals?
(i.e. How much C?)

Cash holding

How many withdrawals?
(i.e. How much C?)

Cash holding

High $i \cdot \frac{C}{2}$ but low $b \cdot \frac{T}{C}$

VS

Low $i \cdot \frac{C}{2}$ but high $b \cdot \frac{T}{C}$
How many withdrawals?
(i.e. How much C?)

Cash holding

\[ C \]
\[ \frac{C}{2} \]

Time

High \( i \cdot \frac{C}{2} \) but low \( b \cdot \frac{T}{C} \)

VS

Cash holding

\[ \frac{c}{C} \]
\[ \frac{c}{2} \]

Time

Low \( i \cdot \frac{C}{2} \) but high \( b \cdot \frac{T}{C} \)

?
Cost minimization

\[ TC(C) = b \frac{T}{C} + i \frac{C}{2} \]

\[ \frac{\partial TC(C)}{\partial C} = 0 \]

\[ -b \frac{T}{C^2} + \frac{i}{2} = 0 \]

\[ C^* = \sqrt{\frac{2bT}{i}} \]

(and we also check that:)

\[ \frac{\partial^2 TC(C)}{\partial C^2} = \frac{bT}{2C^3} > 0 \]
The optimal $C^*$

\[\text{total cost} = i \frac{C}{2} + b \frac{T}{C}\]

\[\text{interest cost} = i \frac{C}{2}\]

\[\text{“brokers’ fees”} = b \frac{T}{C}\]

\[C^* = \sqrt{\frac{2bT}{i}} = f(T, b, i)\]
What if:  «Brokers’ fees» = \( b + (k \cdot C) \) ?

(i.e. what if fees vary with C?)
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square root formula still holds!

\[
C^* = \sqrt{\frac{2bT}{i}}
\]
Tertium datur: third interpretation

• What if receipts *precede* expenditures?
  → Possibility to withhold cash in $t = 0$

• New assumptions:

$$T = R + I$$

«Brokers’ fees» for

\[ \text{withdrawing: } b_w + k_w C \]
\[ \text{investing: } b_d + k_d C \]

(= depositing)
Tertium datur: third interpretation

• What if receipts *precede* expenditures?
  → Possibility to withhold cash in \( t = 0 \)

• New assumptions:

\[ T = (R + I) \]

- Euros withheld in \( t = 0 \)
- (\( R \) = remainder)
- «Brokers’ fees» for withdrawing:

\[ b_w + k_w C \]

- Investing:

\[ b_d + k_d C \]

\( (= \text{depositing}) \)
The (new) cost function (1/3)

1) Opportunity cost of withholding $R$ euros in $t = 0$:

$$i \frac{R}{2} \cdot \frac{R}{T} = i \frac{T - I}{2} \cdot \frac{T - I}{T}$$

2) «Brokers’ fees» for investing $I$ euros in $t = 0$:

$$b_d + k_d I$$
The (new) cost function (1/3)

1) Opportunity cost of withholding \( R \) euros in \( t = 0 \):

\[
\frac{i}{2} \cdot \frac{R}{T} = i \frac{T - I}{2} \cdot \frac{T - I}{T}
\]

average cash holding between \( t = 0 \) and \( t = R / T \)

2) «Brokers’ fees» for investing \( I \) euros in \( t = 0 \):

\[ b_d + k_d I \]
The (new) cost function (2/3)

3) «Brokers’ fees» for withdrawing the invested cash:

\[(b_w + k_w C) \frac{I}{C}\]

4) Opportunity cost of cash withdrawn:

\[i \frac{C}{2} \cdot \frac{I}{T}\]
The (new) cost function (2/3)

3) 「Brokers’ fees」 for withdrawing the invested cash:

\[(b_w + k_w C) \frac{I}{C}\]

number of withdrawals

4) Opportunity cost of cash withdrawn:

\[i \frac{C}{2}, \frac{I}{T}\]

time left after\nt = R / T

average cash holding after\nt = R / T
The (new) cost function (3/3)

1+2+3+4) Total cost function:

\[
TC(C, I) = \left[ i \frac{T - I}{2} \cdot \frac{T - I}{T} \right]_1 + (b_d + k_d I) \left[ \right]_2 + (b_w + k_w C) \frac{I}{C} \left[ \right]_3 + i \frac{C}{2} \cdot \frac{I}{T} \left[ \right]_4
\]
Cost minimization: $C^*$ and $R^*$

\[ \frac{\partial TC(C)}{\partial C} = 0 \Rightarrow C^* = \sqrt{\frac{2b_wT}{i}} \]

\[ \frac{\partial TC(C)}{\partial I} = 0 \Rightarrow -i \frac{T - I}{T} + k_d + \frac{b_w}{C} + k_w + \frac{Ci}{2T} = 0 \]

Substituting $T - I = R$, we obtain:

\[ R = \frac{C}{2} + \frac{b_wT}{Ci} + \frac{T(k_d + k_w)}{i} \]

\[ = \frac{C}{2} \Rightarrow R^* = C + \frac{T(k_d + k_w)}{i} \]
Why does this model matter for the macroeconomy?

I. Demand for cash in stationary economies can be $\neq 0$

II. Demand for cash can rise *less than* in proportion with the volume of transactions

III. Transaction can rise *more than* in proportion with demand for cash, i.e. «*the effect on real income of an injection of cash may have been underestimated*»

IV. This model provides support to the so-called Pigou effect
Why does this model matter for the macroeconomy?

• Baumol’s comment to the model’s assumptions in part III of the paper

• Rational behavior assumption: does it hold?
  • Akerlof, Shiller (2015) “Phishing for Phools: The Economics of Manipulation and Deception”

Source: