Baumol’s transactions demand for cash

A succint summary of Baumol’s paper and why it matters for the macro-economy

Presentation by Nicolò Carnevale

The Transactions Demand for Cash: An Inventory Theoretic Approach
Author(s): William J. Baumol
Source: *The Quarterly Journal of Economics*, Vol. 66, No. 4 (Nov., 1952),
Published by: Oxford University Press
Stable URL: http://www.jstor.org/stable/1882104
Accessed: 29-10-2016 01:45 UTC

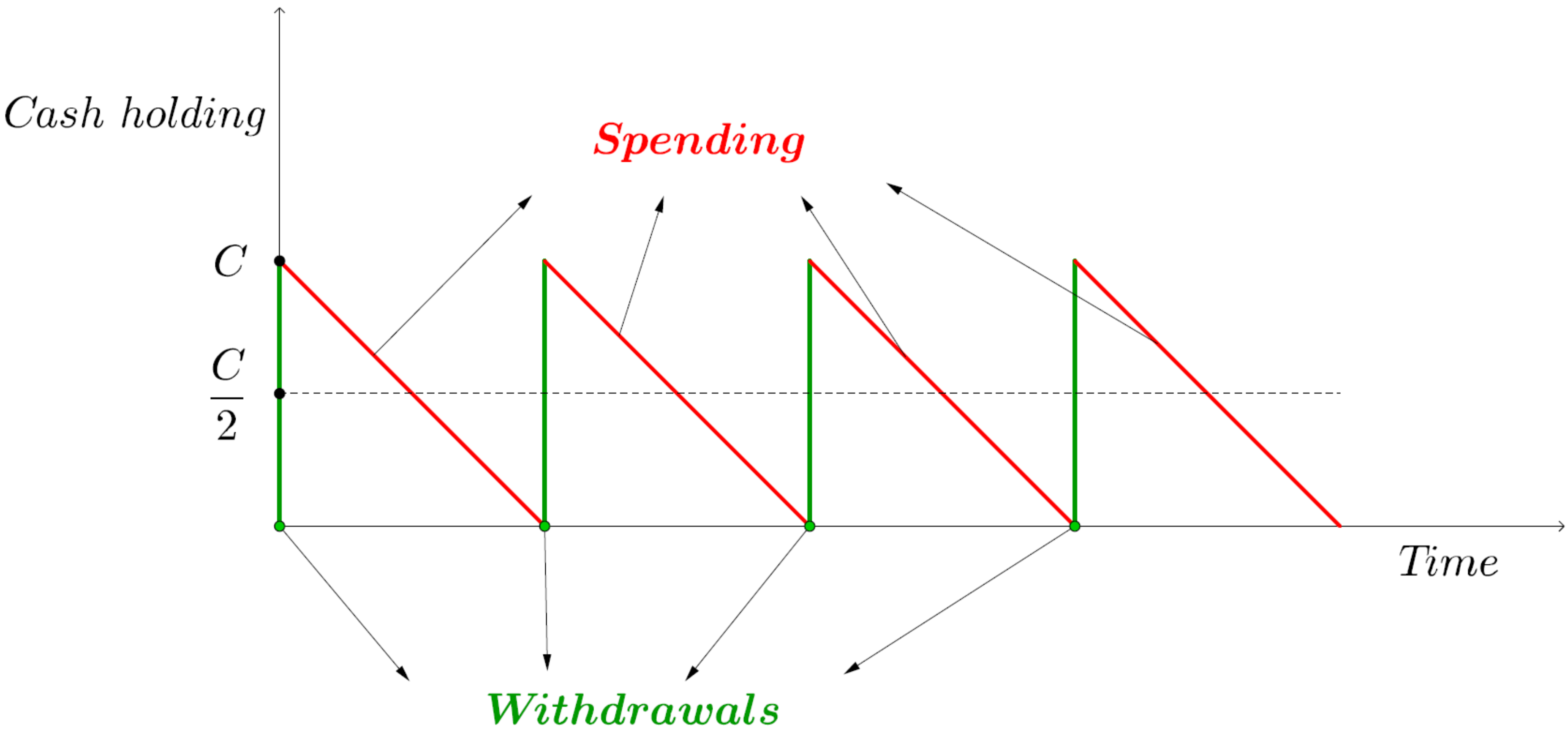
Assumptions

- One individual
- Rational behaviour → cost minimization
- T = payment in € in a steady stream
- i =

  - interest cost in € (borrowing money)

  - or, opportunity cost in € (withdrawal from investment)
- b = "brokers' fees"
- C = amount of cash borrowed/withdrawn

For example:



The cost function

1) Cost of «Brokers' fees»:

$$b \cdot \frac{T}{C}$$

2) Interest cost of holding cash:

$$i \cdot \frac{C}{2}$$

The cost function

1) Cost of «Brokers' fees»:

$$b \cdot \frac{T}{C}$$ number of withdrawals

2) Interest cost of holding cash:

$$i \cdot \frac{C}{2}$$ average cash holding

The cost function

1) Cost of «Brokers' fees»:

$$b \cdot \frac{T}{C}$$ number of withdrawals

2) Interest cost of holding cash:

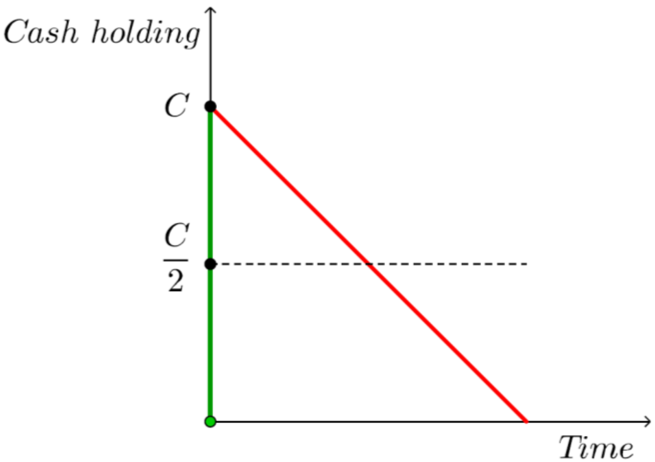
$$i \cdot \frac{C}{2}$$ average cash holding

1+2) Total cost function:

$$TC(C) = b\,\frac{T}{C} + i\,\frac{C}{2}$$

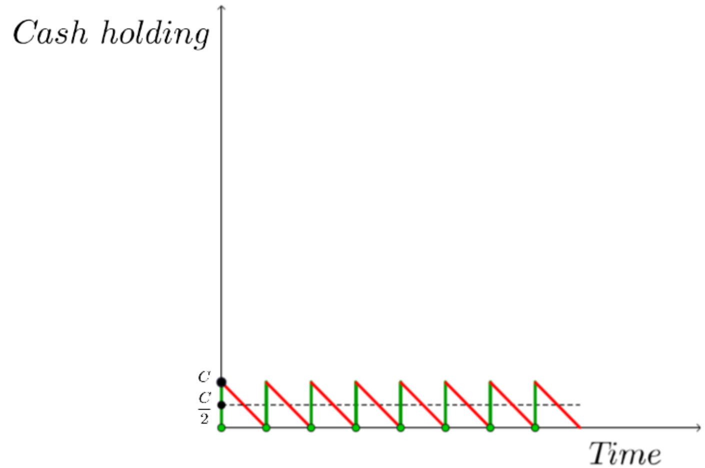
How many withdrawals?

(i.e. How much C?)



High $i \cdot \dfrac{C}{2}$ but **low** $b \cdot \dfrac{T}{C}$

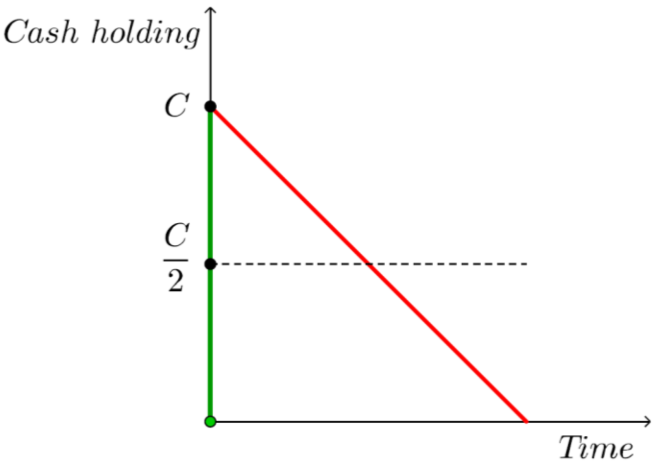
VS



Low $i \cdot \dfrac{C}{2}$ but **high** $b \cdot \dfrac{T}{C}$

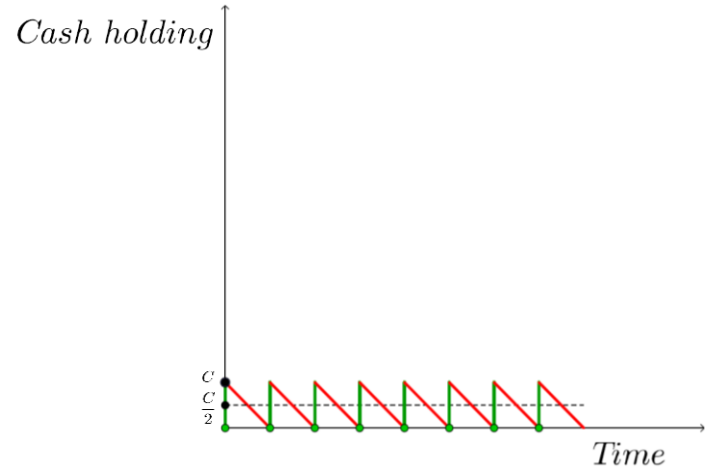
How many withdrawals?

(i.e. How much C?)

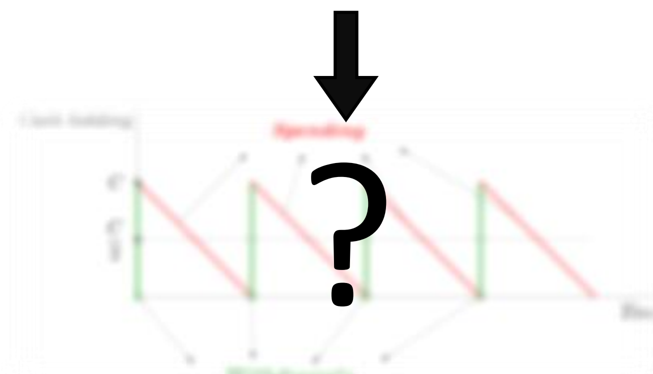


High $i \cdot \frac{C}{2}$ but **low** $b \cdot \frac{T}{C}$

VS



Low $i \cdot \frac{C}{2}$ but **high** $b \cdot \frac{T}{C}$



Cost minimization

$$TC(C) = b\,\frac{T}{C} + i\,\frac{C}{2}$$

$$\frac{\partial TC(C)}{\partial C} = 0$$

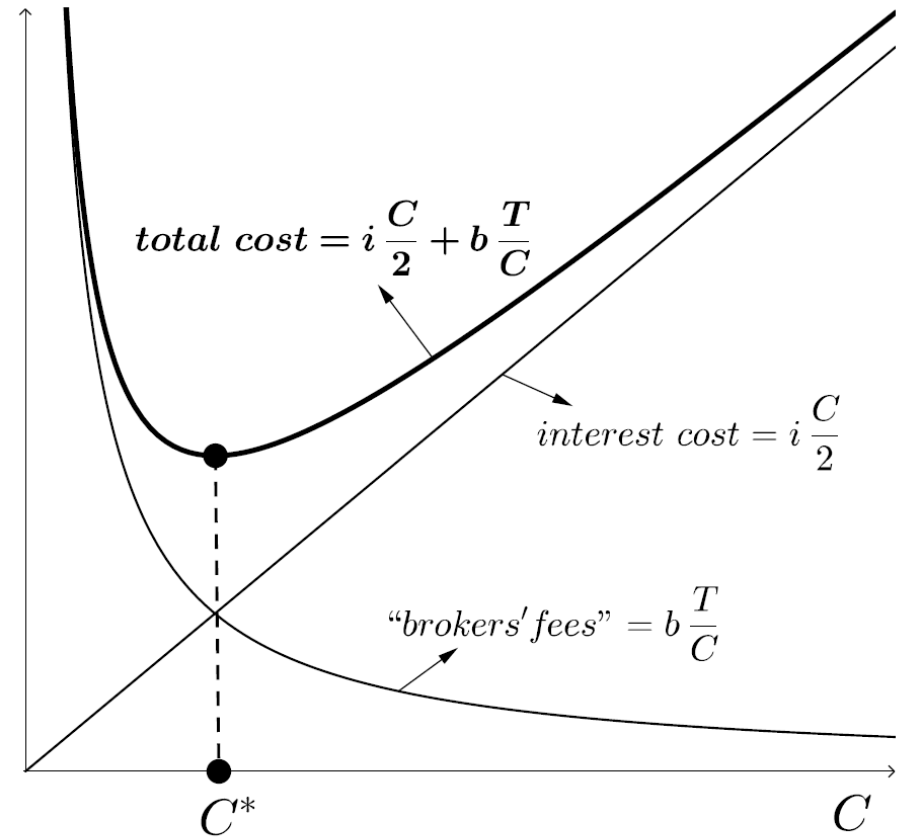
$$-b\,\frac{T}{C^2} + \frac{i}{2} = 0$$

$$C^* = \sqrt{\frac{2bT}{i}}$$

(and we also check that:)

$$\frac{\partial^2 TC(C)}{\partial C^2} = \frac{bT}{2C^3} > 0$$

The optimal C^*



$$C^* = \sqrt{\frac{2bT}{i}} = f(\underset{+}{T}, \underset{+}{b}, \underset{-}{i})$$

What if: «Brokers' fees» = $b + (k \cdot C)$?

(i.e. what if fees vary with C?)

What if: «Brokers' fees» = $b + \underline{\underline{(k \cdot C)}}$?

(i.e. what if fees vary with C?)

square root formula still holds!

$$C^* = \sqrt{\frac{2bT}{i}}$$

Tertium datur: third interpretation

- What if receipts *precede* expenditures?
  → Possibility to withhold cash in $t = 0$
- New assumptions:

$$T = R + I$$

«Brokers' fees» for

withdrawing: $b_w + k_w C$

investing: $b_d + k_d C$

(= **d**epositing)

Tertium datur: third interpretation

- What if receipts *precede* expenditures?
  → Possibility to withhold cash in $t = 0$
- New assumptions:

euros invested in $t = 0$

$$T = R + I$$

euros withheld in $t = 0$
(R = remainder)

«Brokers' fees» for → **w**ithdrawing: $b_w + k_w C$
→ investing: (= **d**epositing) $b_d + k_d C$

The (new) cost function (1/3)

1) Opportunity cost of withholding R euros in *t* = 0 :

$$i\,\frac{R}{2}\cdot\frac{R}{T} = i\,\frac{T-I}{2}\cdot\frac{T-I}{T}$$

2) «Brokers' fees» for investing I euros in t = 0:

$$b_d + k_d I$$

The (new) cost function (1/3)

1) Opportunity cost of withholding R euros in $t = 0$:

$$i\frac{R}{2}\cdot\frac{R}{T} = i\,\frac{T-I}{2}\cdot\frac{T-I}{T}$$

average cash holding between $t = 0$ and $t = R / T$

time needed to run out of R

2) «Brokers' fees» for investing I euros in t = 0:

$$b_d + k_d I$$

The (new) cost function (2/3)

3) «Brokers' fees» for withdrawing the invested cash:

$$(b_w + k_w C)\,\frac{I}{C}$$

4) Opportunity cost of cash withdrawn:

$$i\,\frac{C}{2}\cdot\frac{I}{T}$$

The (new) cost function (2/3)

3) «Brokers' fees» for withdrawing the invested cash:

$$(b_w + k_w C)\frac{I}{C}$$

number of withdrawals

4) Opportunity cost of cash withdrawn:

$$i\frac{C}{2}\cdot\frac{I}{T}$$

average cash holding after $t = R / T$

time left after $t = R / T$

The (new) cost function (3/3)

1+2+3+4) Total cost function:

$$TC(C,I) = \underbrace{i\,\frac{T-I}{2}\cdot\frac{T-I}{T}}_{1} + \underbrace{(b_d + k_d I)}_{2} + \underbrace{(b_w + k_w C)\,\frac{I}{C}}_{3} + \underbrace{i\,\frac{C}{2}\cdot\frac{I}{T}}_{4}$$

Cost minimization: C^* and R^*

- $\dfrac{\partial TC(C)}{\partial C} = 0 \;\Rightarrow C^* = \sqrt{\dfrac{2b_w T}{i}}$
- $\dfrac{\partial TC(C)}{\partial I} = 0 \;\Rightarrow\; -i\,\dfrac{T-I}{T} + k_d + \dfrac{b_w}{C} + k_w + \dfrac{Ci}{2T} = 0$

Substituting $T - I = R$, we obtain:

$$R = \frac{C}{2} + \underbrace{\frac{b_w T}{Ci}}_{=\frac{C}{2}} + \frac{T(k_d + k_w)}{i}$$

$$\Rightarrow\; R^* = C + \frac{T(k_d + k_w)}{i}$$

Why does this model matter for the macroeconomy?

I. Demand for cash in stationary economies can be $\neq 0$
II. Demand for cash can rise *less than* in proportion with the volume of transactions
III. Transaction can rise *more than* in proportion with demand for cash, i.e. «*the effect on real income of an injection of cash may have been underestimated*»
IV. This model provides support to the so-called Pigou effect

Why does this model matter for the macroeconomy?

- Baumol’s comment to the model’s assumptions in part III of the paper

- Rational behavior assumption: does it hold?
  - Akerlof, Shiller (2015) “Phishing for Phools: The Economics of Manipulation and Deception”

- D. Romer, “A Simple General Equilibrium Version of the Baumol-Tobin Model”, The Quarterly Journal of Economics, Vol. 101, No. 4 (Nov., 1986), pp. 663-686

Source:

- Baumol, William J. “The Transactions Demand for Cash: An Inventory Theoretic Approach. The Quarterly Journal of Economics 66.4 (1952): 545