The Economics of Uncertainty

Lecture 5
Econ 4905, Fall 2016
The Economics of Uncertainty

• The state of nature: $S$

• Realizations: $s_1, s_2, ...$

• Intrinsic Uncertainty
  • Random fundamentals
  • Examples
    $$s_1 = \text{rain}, \quad s_2 = \text{drought}$$
    $$s_1 = \text{hot}, \quad s_2 = \text{cold}$$

• Extrinsic Uncertainty
  • Randomness that does not effect the fundamentals, but does affect outcomes
  • Examples
    $$s_1 = \text{no run}, \quad s_2 = \text{run}$$
    $$s_1 = \text{sunspots}, \quad s_2 = \text{no sunspots}$$
Expected Utility

• von Neumann and Morgenstern
  • Expected Utility
    • $V = \pi(s_1)u(x(s_1)) + \pi(s_2)u(x(s_2))$
    • $\pi(s_1) = 1 - \pi(s_2)$
    • $V = \int u(x(s))\pi(s)ds$
  • Risk aversion:
    • $u(x)$
    • $u'(x) > 0$
    • $u''(x) < 0$ Risk-averse
  • Risk-neutral
    • $u''(x) = 0$
  • Risk-loving
    • $u''(x) > 0$
Arrow-Debreu

• Isomorphism
  • Contingent-claims
  • Securities
$F$ – Fire,  $N$ – no fire,  $E$ – expected value

\[ u(x) = \pi(F)u(F) + \pi(N)u(N) \]
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- **CRRA**
  - Kenneth Arrow
  - John Pratt
  - \( u(c) = \frac{c^{1-\gamma}}{1-\gamma} \)
  - For \( \gamma = 1 \), define \( u(c) = \log(c) \)
  - \( u'(c) = \left( \frac{1-\gamma}{1-\gamma} \right) c^{-\gamma} = c^{-\gamma} > 0 \)
  - \( u''(c) = -\gamma c^{-\gamma-1} < 0 \)
- **Risk-aversion**
  - \(- \frac{cu''(c)}{u'(c)} = \frac{\gamma c^{-\gamma-1} \cdot c}{c^{-\gamma}} = \frac{\gamma c^{-\gamma}}{c^{-\gamma}} = \gamma \)
Profit Maximization

- **NOT** an axiom
- Theorem, requiring assumptions
- Perfect markets
Robinson Crusoe

PPF

Fish

Coconuts
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Robinson Crusoe

$u = \text{const.}$

Supporting Price Ratio

$Fish$ $Coconuts$
Robinson Trades

Graph showing the trade-off between Fish and Coconuts.
• Produces to market. Profit max
  • In order to max utility

• Dynamic extension
  • Max PDV
  • If borrowing and lending are perfect

• Uncertainty extension
  • Max contingent-claim profit
  • If insurance is perfect