three periods: \( T = 0, 1, 2 \)

- a single good

- a continuum of agents with measure 1

- Each agent is endowed with 1 unit of the good in period 0.
The Model: Asset Return

\[ T = 0 \quad T = 1 \quad T = 2 \]

\[
-1 \quad \begin{cases} 0 \\ 1 \end{cases} \quad R \\
0
\]
The Model: Preferences

- In period 0, all agents are identical.
- In period 1, some agents become “patient” and others become “impatient”. (private information)
  \[
  \begin{align*}
  u(c_1) & \quad \text{if impatient} \\
  u(c_2) & \quad \text{if patient}
  \end{align*}
  \]
- The probability of being impatient is \( \lambda \) for each agent in period 0.
Autarky

- autarky:
  - utility of the impatient in period 1: \( u(1) \)
  - utility of the patient in period 2: \( u(R) \)
  - expected utility in period 0: \( \lambda u(1) + (1 - \lambda) u(R) \)

- \( 1 < R \)
  - “insurance” against the liquidity shock is desirable.
Banking Economy

- Banks offers demand deposit contract \((d_1, d_2)\).
- Agents
  - make deposits in period 0.
  - withdraw \(d_1\) in period 1.
  - or withdraw \(d_2\) in period 2.
- Free-entry banking sector: \((d_1, d_2)\) maximizes the depositor’s expected utility.
Optimal Deposit Contract

\[
\max_{d_1, d_2} \lambda u(d_1) + (1 - \lambda) u(d_2)
\]

s.t. \[ (1 - \lambda) d_2 \leq (1 - \lambda d_1) R \quad (RC) \]

\[ d_1 \leq d_2 \quad (IC) \]
Optimal Deposit Contract:

\[(1 - \lambda) d_2 = (1 - \lambda d_1) R\]

slope = \[\frac{-\lambda}{1 - \lambda} R\]
Optimal Deposit Contract:

$$\lambda u(d_1) + (1 - \lambda) u(d_2) = \text{const}$$

slope = \(-\frac{\lambda}{1 - \lambda} \frac{u'(d_1)}{u'(d_2)}\)

\((1 - \lambda) d_2 = (1 - \lambda d_1) R\)

slope = \(-\frac{\lambda}{1 - \lambda} R\)

$$\frac{\lambda u'(d_1^*)}{1 - \lambda u'(d_2^*)} = \frac{\lambda R}{1 - \lambda}$$
What do banks do?

- \( u'(d_1^*) / u'(d_2^*) = R \)
- \( u'' < 0 \Rightarrow d_1^* < d_2^* \)
- **CRRA:** \( u(c) = \frac{c^{1-\gamma}}{1-\gamma} \)
  - \( u'(c) = c^{-\gamma} \Rightarrow u'(d_1) / u'(d_2) = (d_2 / d_1)^\gamma \)
  - if \( \gamma = 1 \Rightarrow d_1^* = 1, d_2^* = R \)
  - if \( \gamma > 1 \Rightarrow 1 < d_1^* < d_2^* < R \)
Why do bank runs occur?

- $\gamma > 1 \implies 1 < d_1^* < d_2^* < R$
- IC: $d_1 \leq d_2$
- Expectation: Only the impatient depositors withdraw in period 1.
- A patient depositor can
  \[
  \begin{cases} 
  \text{get } d_2^* & \text{if he withdraws in period 2} \\
  \text{get } d_1^* & \text{if he withdraws in period 1}
  \end{cases}
  \]
Why do bank runs occur?

- $\gamma > 1 \implies 1 < d_1^* < d_2^* < R$

- Expectation: All other depositors demand withdraw in period 1.

- A patient depositor can
  \[
  \begin{cases}
  \text{get nothing} & \text{if he withdraws in period 2} \\
  \text{get } d_1^* \text{ w.p. } (1/d_1^*) & \text{if he withdraws in period 1}
  \end{cases}
  \]