Economics 6130-2
Macroeconomics II
Fall 2016
Problem Set 1 Solutions

1 Money Taxation

Consider an economy with a single commodity, $\ell = 1$, chocolate. There are 5 consumers, so $n = 5$. The endowments are defined as

$$\omega = (\omega_1, \omega_2, \omega_3, \omega_4, \omega_5)$$
$$= (50, 40, 30, 20, 10)$$

1.1 A Single Currency

There is one money. The chocolate price of money is $P^m \geq 0$. In each of the following cases, solve for the set $\mathscr{P}^m$ of equilibrium prices P^m, given the following tax policies τ. Provide the units in which the variables are measured.

a) $\tau = (\tau_1, \tau_2, \tau_3, \tau_4, \tau_5) = (1, 1, 0, 0, -2)$

a) Solution:
In general,

$$x_h = \omega_h - \tau_h P^m > 0$$

Taxes τ are in money, but Mr. h's endowment ω_h is in chocolate. The price of money, P^m, is the rate of exchanging one unit of currency (like dollars) with a unit of real good, here chocolate; P^m is therefore in *chocolate/dollars*.

For Mr. 1, we have $50 - P^m > 0$. Therefore $P^m < 50$.

For Mr. 2, $40 - P^m > 0$. Therefore $P^m < 40$.

Thus, we have $\mathscr{P}^m = [0, 40)$. Note that a worthless currency, $P^m = 0$, is an equilibrium outcome.

b) $\tau = (10, 5, 0, -8, -7)$

b) Solution:
For Mr. 1, $50 - 10P^m > 0$, so $P^m < 5$.
For Mr. 2, $40 - 5P^m > 0$. Thus, $P^m < 8$.
We have $\mathscr{P}^m = [0, 5)$.

c) $\tau = (20, 2, 1, -2, -20)$

c) Solution:
We may immediately note that $\sum_h \tau_h = 20 + 2 + 1 - 2 - 20 = 1 \neq 0$. Thus, taxes are not balanced in this finite economy. The equilibrium price of money must therefore be $\mathscr{P}^m = \{0\}$, such that taxes fail to be bonafide as well. The result will be an economy in autarky, as money will be worthless.

1.2 Two Monies

Consider a scenario where there are 2 monies, red dollars R and blue dollars B, with respective chocolate prices of money, $P^R \geq 0$ and $P^B \geq 0$.

In each of the following cases, solve for the equilibrium exchange rate between B and R. Do these depend on the endowments ω? Give the economic explanation for your answer.

a) $\tau^R = (1, 1, 1, 0, -2)$ and $\tau^B = (1, 0, 0, 0, -2)$

a) Solution:

Recalling that $x_h = \omega_h - P^R\tau_h^R - P^B\tau_h^B$, we may rearrange the equation to get

$$x_h - \omega_h = -P^R\tau_h^R - P^B\tau_h^B$$

If we sum over h consumers, we get

$$\sum_h (x_h - \omega_h) = -P^m \sum_h \tau_h^R - P^m \sum_h \tau_h^B$$

And since when markets clear, $\sum_h (x_h - \omega_h) = 0$,

$$P^R \sum_h \tau_h^R + P^B \sum_h \tau_h^B = 0 \quad \Rightarrow \quad P^R \sum_h \tau_h^R = -P^B \sum_h \tau_h^B$$

Rearranging further, we get the exchange rate as

$$\frac{P^R}{P^B} = -\frac{\sum_h \tau_h^B}{\sum_h \tau_h^R}$$

In this case, $\sum_h \tau_h^R = 1 + 1 + 1 - 2 = 1$, while $\sum_h \tau_h^B = 1 - 2 = -1$, so

$$\frac{P^R}{P^B} = -\left(\frac{-1}{1}\right) = 1$$

Of course, this is also equivalent to $\frac{P^B}{P^R} = 1$ as well.

b) $\tau^R = (1, 1, 0, -1, -2)$ and $\tau^B = (1, 1, 1, 0, -2)$

b) Solutions:

Here, $\sum_h \tau_h^R = 1 + 1 - 1 - 2 = -1$, while $\sum_h \tau_h^B = 1 + 1 + 1 - 2 = 1$. Thus, it again holds that $\frac{P^R}{P^B} = -\left(\frac{-1}{1}\right) = 1$ (and exchanging in the other direction, $\frac{P^B}{P^R} = 1$).

c) $\tau^R = (3, 2, 1, 0, -6)$ and $\tau^B = (4, 0, -1, -1, -2)$

c) Solutions:

Finally, we have $\sum_h \tau_h^R = 3 + 2 + 1 - 6 = 0$, while $\sum_h \tau_h^B = 4 - 1 - 1 - 2 = 0$. The exchange rate is therefore indeterminate, as $\frac{P^R}{P^B} = \frac{0}{0}$ is not well-defined.

These exchange rates are independent of the endowments ω; the supply and demand for the currencies completely determines the exchange rate between them unless one or both currencies are worthless. If both tax policies are balanced, then the exchange rate is indeterminate since there are no currency trades.

1.3 The Absence of Money Illusion

Explain the difference between the “absence of money illusion” and the “quantity theory of money”. Be precise (with symbols).

Solution:

Taxes only matter through their real values. Only the term $P^m \tau_h$ matters to Mr. h.

Absence of money illusion: Let P^m be an equilibrium price of money given the tax vector τ. If the tax vector is multiplied by some scalar λ to become $\lambda\tau$, then $\frac{P^m}{\lambda}$ is *an* equilibrium price of money. In other words, if $\mathscr{P}^m = [0, \bar{P}^m]$ when the tax vector is τ, then when the tax vector is $\lambda\tau$, it follows that $\mathscr{P}^m = \left[0, \frac{\bar{P}^m}{\lambda}\right)$.

Quantity theory of money: If P^m is an equilibrium price of money when the tax vector is τ, then when taxes become $\lambda\tau$, the equilibrium price of money becomes $\frac{P^m}{\lambda}$.

The quantity theory of money is true if and only if people believe it to be true, while the absence of money illusion is a statement about sets.

In other words, if outside money is doubled, then under the quantity theory of money, the price of money will halve (and the price level for real goods, by extension, will double). In contrast, with an absence of money illusion, it is only a *possibility* that the same fiscal policy change will halve the price of money and double the price level. The actual price of money and price level after the tax regime change, however, will be indeterminate.

As noted in lecture, our models are consistent with the AMI, but not strictly with QTM.