1 Overlapping Generations, Part I

Consider the following OLG economy:

2-period lives.
1 commodity per period, \( l = 1 \).
Stationary endowments:
\[
\begin{align*}
\omega^0_t &= B > 0 \text{ for } t = 0 \\
(\omega^t_t, \omega^{t+1}_t) &= (A, B) >> 0 \text{ for } t = 1, 2, ...
\end{align*}
\]

Stationary preferences:
\[
\begin{align*}
u^0_0(x^0_0) &= D \log(x^0_0) \text{ for } t = 0 \\
u^t_t(x^t_t, x^{t+1}_t) &= C \log(x^t_t) + D \log(x^{t+1}_t) \text{ for } t = 1, 2, ...
\end{align*}
\]

1 person per generation
Passive fiscal policy:
\[
m^1_0 = 2 \quad m^s_t = 0 \text{ otherwise}
\]

Goods price of money is \( p^m \geq 0 \).

For each of the following cases: Calculate the offer curve for Mr. \( t \geq 1 \).
Then find and plot the offer curve in excess demand space \((z^t, z^{t+1})\), or equivalently in the \((x^t_t - \omega^t_t, x^{t+1}_t - \omega^{t+1}_t)\) domain. Plot the reflected offer curve, and analyze the global dynamics.
Is there a pattern?

Derive the conditions on the MRS for a “Samuelson” versus a “Classical” (or “Ricardo”) economy and relate them to the above.

2 Overlapping Generations, Part II

Consider the following OLG economy:
Pure exchange, 2-period lives, one consumer per generation.

\[
\begin{align*}
    u_0(x^1_0) &= x^1_0 \\
    \omega^1_0 &= 1, \text{ for } t = 0, \\
    u_t(x^t_t, x^{t+1}_t) &= x^t_t + x^{t+1}_t \\
    (\omega^t_t, \omega^{t+1}_t) &= (1, 1) \text{ for } t = 1, 2, \ldots
\end{align*}
\]

Money transfers:

\[
\begin{align*}
    m^0_1 &= 2, m^1_1 = -1, \\
    m^2_t &= 1, m^s_t = 0 \text{ otherwise}.
\end{align*}
\]

(a) What is the non-monetary equilibrium allocation? What are the prices? What are the interest rates?

(b) Derive the reflected offer curve for consumer \( t = 1, 2, \ldots \)

(c) Derive the set of equilibrium money prices.

(d) Draw the phase diagram and show the full evolution of this economy (depending on the price of money).

(e) What is the Pareto optimal allocation associated with the above (money) tax-transfer policy?
(f) Find an alternative tax-transfer policy and associated allocation which is not Pareto optimal but in which everyone is strictly better off than they would be in autarky.

(g) Find an alternative tax-transfer policy and associated allocation which is Pareto optimal and in which everyone is strictly better off than they would be in the non-monetary equilibrium.