Economics 4905

Fall 2016

Cornell University

Financial Fragility and the Macro-economy

Prelim #1 (Solutions)

Monday, October 17, 2016

2:55 PM to 4:10 PM

291 Statler Hall

Answer each of the 3 questions. The prelim is designed to be done in 60 minutes, but you are permitted to use up to 75 minutes. Do not consult anyone or anything during the prelim. Do not use calculators, computers, or any other electronic devices. Use black ink or black pencil, if possible. Leave all personal items in places designated by the proctor. Show your work. There is no need to simplify your arithmetic answers.

1. Outside Money Taxes (20 minutes)

1.a) 5 people. Endowments $\omega = (\omega_1, \omega_2, \omega_3, \omega_4, \omega_5) = (900, 800, 700, 600, 500)$.
Find the set of equilibrium money prices $\mathscr{P}^m$ when money taxes $\tau = (\tau_1, \tau_2, \tau_3, \tau_4, \tau_5)$ are given by

1.a.i) $\tau = (2, 2, 0, -2, -2)$

Solution:

$\sum_h \tau_h = 2 + 2 + 0 - 2 - 2 = 0$, so taxes are balanced and therefore bonafide.

$$900 - 2P^m > 0 \quad \Rightarrow \quad P^m < 450$$

$$800 - 2P^m > 0 \quad \Rightarrow \quad P^m < 400$$

So $\mathscr{P}^m = [0, 400)$.

1.a.ii) $\tau = (2, 2, -1, -3, -2)$

Solution:

$\sum_h \tau_h = 2 + 2 - 1 - 3 - 2 = -2$, so taxes are *not* balanced and hence are *not* bonafide. So, $\mathscr{P}^m = \{0\}$.

1.a.iii) $\tau = (0, 0, 0, 0, 0)$

Solution:

$\sum_h \tau_h = 0 + 0 + 0 + 0 + 0 = 0$, so taxes are balanced and hence bonafide. P^m is indeterminate, and $\mathscr{P}^m = [0, \infty)$. No one net buys or net sells currency.

1.b) 3 people, two monies $B\$$ and $R\$$, with money taxes generated by

$$\tau^B = (\tau_1^B, \tau_2^B, \tau_3^B) \text{ and } \tau^R = (\tau_1^R, \tau_2^R, \tau_3^R)$$

Find the exchange rate when

1.b.i) $\tau^B = (2, 2, -2)$ and $\tau_R = (-1, -1, -1)$

Solution:

$\sum_h \tau_h^B = 2$ and $\sum_h \tau_h^R = -3$, so

$$2P^B - 3P^R = 0 \quad \Rightarrow \quad 2P^B = 3P^R$$
$$\frac{P^B}{P^R} = \frac{3}{2}$$

The exchange rate is independent of ω. To do more, however, we must specify ω. As such, it was not necessary to find the set of equilibrium money prices for this question.

1.b.ii) $\tau^B = (2, -1, -1)$ and $\tau_R = (1, 0, -1)$

Solution:

$\sum_h \tau_h^B = 0$ and $\sum_h \tau_h^R = 0$, so as

$$P^B \sum_h \tau_h^B + P^R \sum_h \tau_h^R = 0 \quad \Rightarrow \quad P^B \sum_h \tau_h^B = -P^R \sum_h \tau_h^R$$

$$\frac{P^B}{P^R} = -\frac{\sum_h \tau_h^R}{\sum_h \tau_h^B} = \frac{0}{0}$$

The exchange rate is indeterminate.

1.b.iii) $\tau^B = (1, 1, 1)$ and $\tau_R = (-1, -1, -1)$

Solution:

$\sum_h \tau_h^B = 1$ and $\sum_h \tau_h^R = -3$

$$3P^B - 3P^R = 0 \quad \Rightarrow \quad \frac{P^B}{P^R} = 1$$

1.b.iv) $\tau^B = (5, 0, 0)$ and $\tau_R = (0, 0, -10)$

Solution:

$\sum_h \tau_h^B = 5$ and $\sum_h \tau_h^R = -10$

$$5P^B - 10P^R = 0 \quad \Rightarrow \quad \frac{P^B}{P^R} = \frac{10}{5} = 2$$

2. The Diamond-Dybvig Bank

The probability of being impatient is 0.50. The utility function is:

$$u(c) = 10 - \frac{1}{(0.5)\sqrt{c}}.$$

The rate of return to the asset harvested late is 400%, i.e., $R = 5$.
The depositor's endowment is $y = 7$, which she deposits in the bank. The banking contract is (d_1, d_2), where $t = 1, 2$; it is the promised withdrawal for depositors seeking to withdraw in period t.

2.a) Graph the following in (d_1, d_2) space:
2.a.i) The resource constraint RC

Solution:

Resource Constraint: $(1-\lambda)d_2 \le (y - \lambda d_1)R$. Therefore,

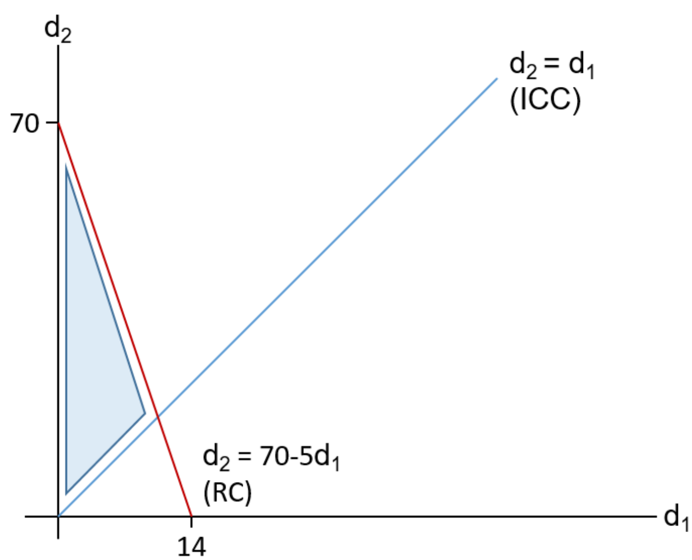
$$\frac{1}{2}d_2 \le \left(7 - \frac{1}{2}d_1\right)5$$

$$d_2 \le 70 - 5d_1 \quad \Rightarrow \quad \text{Slope of the RC} = -5$$

2.a.ii) The incentive compatibility constraint IC (or ICC)

Solution:

The ICC is $d_2 \ge d_1$. Altogether, we have



2.b) What is the depositor's *ex-ante* expected utility W as a function of c_1, consumption in period 1, and c_2, consumption in period 2? Show this in the (d_1, d_2)-space graph.

Solution:

The ex-ante expected utility of the depositor will be

$$W(c_1, c_2) = \frac{1}{2}u(c_1) + \frac{1}{2}u(c_2) = 10 - c_1^{-1/2} - c_2^{-1/2}$$

Isoquant for W: (A level set of $W = \alpha$)

$$10 - c_1^{-1/2} - c_2^{-1/2} = \alpha$$

This may be expressed as

$$c_2^{-1/2} = 10 - c_1^{-1/2} - \alpha \quad \Rightarrow \quad c_2 = (10 - c_1^{-1/2} - \alpha)^{-2}$$

$$c_2 = \frac{1}{(10 - c_1^{-1/2} - \alpha)^2}$$

This will be a concave-up isoquant with a convex preferred set; its graph will be similar to $y = \frac{1}{x}$, although it will be slightly more complicated. To draw it more precisely, we may note that along the isoquant $W = \alpha$,

$$\left(\frac{dc_2}{dc_1}\right)_{W=\alpha} = -\frac{\frac{\partial W}{\partial c_2}}{\frac{\partial W}{\partial c_2}} = -\frac{c_2^{-3/2}}{c_1^{-3/2}}$$

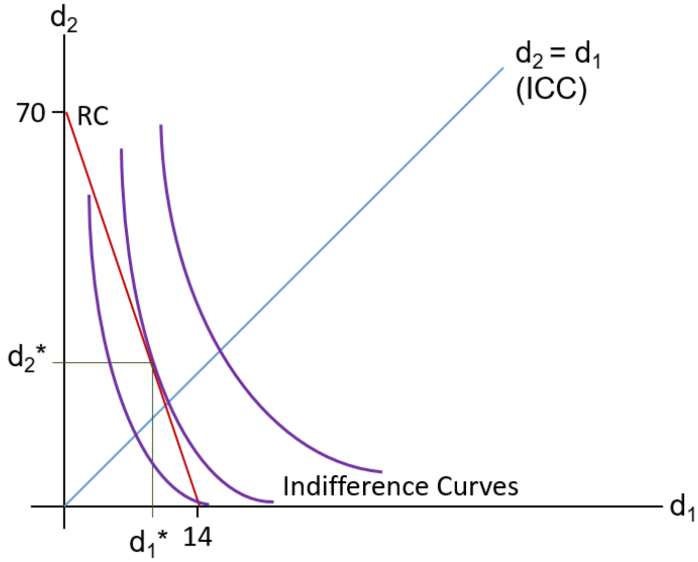
By the implicit function theorem. This leads us to

$$= -\left(\frac{c_1}{c_2}\right)^{3/2} < 0 \ \forall c_1, c_2 \in \mathbb{R}_{++}$$

So the isoquant will be downward-sloping.
For the isoquant to be tangent with the RC,

$$\left(\frac{dc_2}{dc_1}\right)_{W=\alpha,\ RC} = -5$$

This level of mathematical formalism was not explicitly required on the exam. However, it may have helped you draw the curve in the phase space.

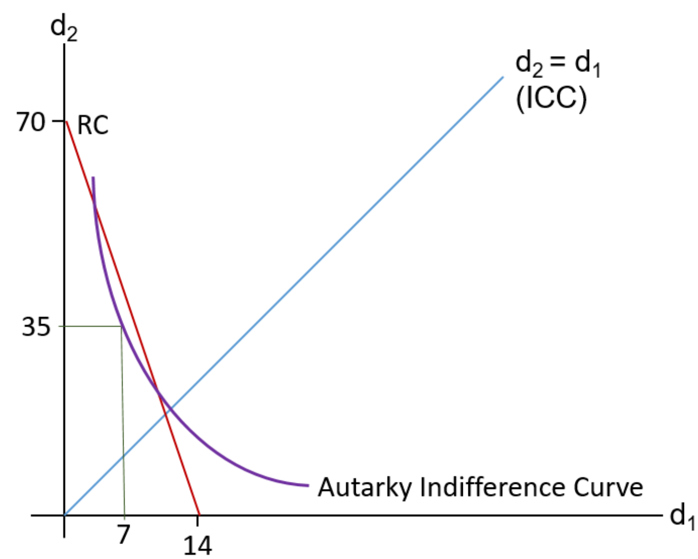


2.c) Solve for the depositor's expected utility in autarky. Show this on the graph in (d_1, d_2) space.

Solution:

In autarky,

$$W(y, yR) = \frac{1}{2}u(7) + \frac{1}{2}u(35) = \frac{1}{2}\left(10 - \frac{2}{\sqrt{7}}\right) + \frac{1}{2}\left(10 - \frac{2}{\sqrt{35}}\right)$$
$$= 10 - \frac{1}{\sqrt{7}} - \frac{1}{\sqrt{35}} \approx 9.47$$



2.d) Solve for the so-called "optimal deposit contract."

To find the optimal contract, we must solve

$$\arg\max_{d_1,d_2} \{W(d_1,d_2) = \lambda u(d_1) + (1-\lambda)u(d_2)\}$$

subject to RC and ICC. By using Lagrangian optimization, or by remembering the optimization's corollary,

$$\frac{u'(d_1)}{u'(d_2)} = R$$

$$\Rightarrow \quad \left(\frac{d_1}{d_2}\right)^{-3/2} = \left(\frac{d_2}{d_1}\right)^{3/2} = 5$$

So

$$\frac{d_2}{d_1} = 5^{2/3} \quad \Rightarrow \quad d_2 = 5^{2/3}d_1 \quad \text{and so} \quad d_1 = 5^{-2/3}d_2$$

Using the RC,

$$d_2 = 70 - 5d_1 \quad \Rightarrow \quad d_2 = 70 - 5(5^{-2/3}d_2) = 70 - 5^{1/3}d_2$$

$$d_2 + 5^{1/3}d_2 = 70 \quad \Rightarrow \quad (1+5^{1/3})d_2 = 70$$

$$d_2^* = \frac{70}{1+5^{1/3}} \approx 25.83$$

And then

$$d_1^* = 5^{-2/3}d_2^* = \frac{(5^{-2/3})(70)}{1+5^{1/3}} \approx 8.83$$

Note that $d_2^* > d_1^*$, consistent with the ICC.

2.e) What is W if there is no run? If there is a run?

Solution:

If there is no run, then the depositor simply consumes the optimal deposit contract withdrawals, d_1^*, d_2^*. Then,

$$W_{no-run} = W(d_1^*, d_2^*) = \frac{1}{2}u(d_1^*) + \frac{1}{2}u(d_2^*)$$

$$= \frac{1}{2}\left(10 - \frac{1}{\sqrt{8.83}}\right) + \frac{1}{2}\left(10 - \frac{1}{\sqrt{25.83}}\right) \approx 9.734$$

Say there is no partial suspension of convertibility in the event of a run. Then let $\hat{\lambda}$ be the fraction of depositors whom the bank can serve.

$$W_{run} = \hat{\lambda}u(d_1^*) + \hat{\lambda}u(0)$$

Since $\lim_{c \to 0} u(c) = -\infty$, we may loosely write

$$W_{run} = -\infty$$

Alternatively, suppose the bank realizes that there is a run going on after 50% of its depositors all arrive at the bank. Then, the bank switches to partial suspension of convertibility. To put this differently, the bank honors half of the depositor's demands for d_1^*, then limits withdrawals to allow for the servicing of the other half of withdrawal requests.

$$\begin{aligned}
W_{run} &= \frac{1}{2}u(d_1^*) + \frac{1}{2}u\left(y - \frac{d_1^*}{2}\right) \\
&= \frac{1}{2}u(8.83) + \frac{1}{2}u\left(7 - \frac{8.83}{2}\right) \\
&= \frac{1}{2}\left[10 - \frac{2}{\sqrt{8.83}}\right] + \frac{1}{2}\left[10 - \frac{2}{\sqrt{7 - \frac{8.83}{2}}}\right] \approx 8.42
\end{aligned}$$

Note how in this solution, the run would do less damage to depositors as a whole than in the first proposed W_{run} outcome, but would still be worse than not having a run at all.

3. Risk and Insurance

Harry, an expected utility maximizer, faces two states of nature, α and β with probabilities $\pi(\alpha) = \frac{9}{10}$ and $\pi(\beta) = \frac{1}{10}$. Harry's consumption in state α is $c(\alpha) = 100$, while in state β his consumption is $c(\beta) = 10$. Harry's expected utility is

$$V = \pi(\alpha)u(c(\alpha)) + \pi(\beta)u(c(\beta))$$

Where $u(c)$ is CRRA with

$$u(c) = 2c^{1/2}$$

3.a) Calculate the first derivative $u'(c)$ and the second derivative $u''(c)$.

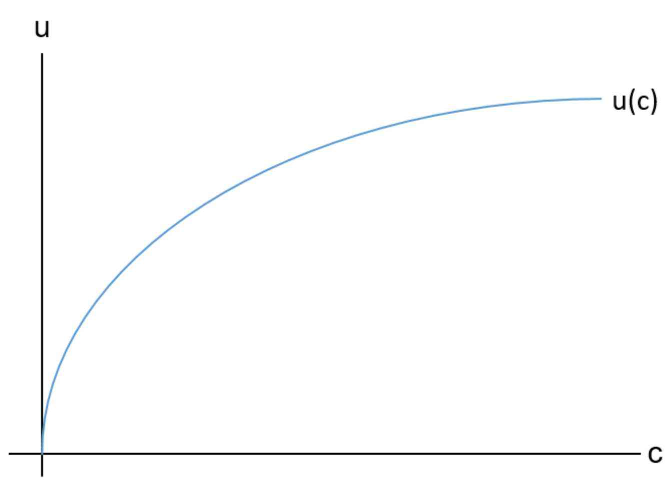
Solution:

$$u'(c) = c^{-1/2} > 0 \ \forall c \in \mathbb{R}_{++}$$
$$u''(c) = -\frac{1}{2}c^{-3/2} < 0 \ \forall c \in \mathbb{R}_{++}$$

Note that $u(0) = 0$ and $\lim_{c\to\infty} u(c) = \infty$, while $\lim_{c\to 0} u'(c) = \infty$ and $\lim_{c\to\infty} u'(c) = 0$.

3.b) Graph u(c).

Solution:
Using Part a), we may plot



As $u(c)$ is strictly monotonically increasing and strictly concave (as evidenced by the derivatives), Harry is risk-averse.

3.c) Compute V without insurance.

Solution:

When uninsured, Harry's ex-ante expected utility is

$$V = \frac{9}{10}(2\sqrt{100}) + \frac{1}{10}(2\sqrt{10})$$

$$= 18 + \frac{1}{5}\sqrt{10} \approx 18.633$$

3.d) What is the value of V if Harry can purchase insurance at "fair odds"?

Solution:
If Harry has "fair" insurance, then he will receive the expected consumption payoff in either state.

$$\bar{c} = \pi(\alpha)c(\alpha) + \pi(\beta)c(\beta)$$
$$= \frac{9}{10}(100) + \frac{1}{10}(10) = 91$$

In such a case,

$$V(\bar{c}, \bar{c}) = \frac{9}{10}u(\bar{c}) + \frac{1}{10}u(\bar{c}) = u(\bar{c}) = 2\sqrt{\bar{c}}$$

$$= 2\sqrt{91} \approx 19.078$$

As

$$u(\bar{c}) = 19.078 > 18.633 = \pi(\alpha)u(c(\alpha)) + \pi(\beta)u(c(\beta))$$

Harry will prefer to be insured than not, in that he will be willing to purchase insurance up to a certain price. Another way of saying this is that as a risk-averse individual,

$$U(E[c]) > E[U(c)]$$

Harry would rather get an average payoff with certainty than receive (on average/in expectation) an uncertain payoff.

3.e) What is the largest insurance premium that Harry will purchase?

Solution:
If Harry has no other options, then he will accept any insurance policy that yields greater utility than in autarky, i.e. with $V \geq 18.633$.

Let $\hat{c}$ be the critical value of consumption defined by

$$u(\hat{c}) = 18.633$$

Harry will therefore buy any policy in which $c > \hat{c}$. Thus,

$$u(\hat{c}) = 2\sqrt{\hat{c}} = 18.633 \quad \Rightarrow \quad \sqrt{\hat{c}} = \frac{1}{2}(18.633)$$

$$\hat{c} = \left(\frac{18.633}{2}\right)^2 \approx 86.793$$

Note that $\hat{c} < \bar{c}$.
The premium would thus be

$$p = E(c) - \hat{c} = \bar{c} - \hat{c} = 91 - 86.793 = 4.207$$