Econ6130-2: Lecture 2

Fall 2016
Review of Lecture 1 (Outside) Money Taxation:

- Bonafide Taxes $\tau$
- Balanced Taxes $\tau$
- Bonafide $\tau \equiv$ Balanced $\tau$
- $l = 1, \mathbb{P}^m$ is interval $[0, \bar{\mathbb{P}}^m)$
- $l > 1, \mathbb{P}^m$ is the union of intervals
- Multiple Currencies
- Problem Set 1
- Simple, finite static model
Enriching the model to include dynamics and uncertainty: Debreu’s isomorphism

- "Debreu" Isomorphism
  - Expand definition of commodity, $x^i_{h,s,t}$
  - Commodity type $i$, state of nature $s$, time $t$
  - Contingent claims
  - Futures Market

- Profit Maximization
  - Theorem
  - Not assumption
  - Diagrams
Futures Market

- t=1,2
- l = 1
- present prices, \((p^1, p^2) = (1, p^2)\)

**CP:**

\[
\max u_h(x^1_h, x^2_h)
\]

e.g. \(u_h(x^1_h, x^2_h) = \phi_h(x^1_h) + \beta_h \phi_h(x^2_h)\)

s.t.

\[
p^1(x^1_h - \omega^1_h) + p^2(x^2_h - \omega^2_h) = 0
\]

**CE:** \((p^1, p^2)\) such that

\[
\sum_x x^t_h = \sum \omega^t_h \text{ for } t = 1, 2
\]

where \(x^t_h\) solves CP for \(t = 1, 2\)
Criticism of Futures Market Interpretation:

- Do we really choose today all our future consumptions?
- Do ordinary people use futures market for personal choices over time?
- Everyone on a "meal plan" for everything?
- FM model is real, i.e., non-financial. More stable, but less realistic?
Inside Money Market for Dynamic Economy

- Spot market at each date, $t = 1, 2$
- Saving and dis-saving through "money-market"
- Rational expectations about future spot prices
- Expectations play no role in FM model
Inside Money, continued

**CP:**

\[
\max u_h(x_h^1, x_h^2) \text{ s.t.}
\]

\[
\begin{align*}
\n p^1 x_h^1 + p^{m1} x_h^{m1} &= p^1 \omega_h^1 \\
 p^2 x_h^2 + p^{m2} x_h^{m2} &= p^2 \omega_h^2 \\
 x_h^{m1} + x_h^{m2} &= 0 \text{ or } x_h^{m2} = -x_h^{m1}
\end{align*}
\]

**CE:**

\((p^1, p^2; p^{m1}, p^{m2})\) s.t.

\[
\sum_h x_h^t = \sum_h \omega_h^t \text{ for } t = 1, 2
\]

\[
\sum_h x_h^{m,t} = 0 \quad t = 1, 2
\]

where \(x_h^t\) and \(x_h^{m,t}\) satisfy CP for \(t = 1, 2\)
Simplifying and substituting

- $p^1(x^1_h - \omega_h^1) = -p^{m1}x^{m1}_h$
- $p^2(x^2_h - \omega_h^2) = p^{m2}x^{m1}_h$
- $x^{m1}_h$ is a slack variable permitting us to combine terms (discuss):
- $p^1(x^1_h - \omega_h^1) + p^2(x^2_h - \omega_h^2) = (p^{m2} - p^{m1})x^{m1}_h$
- In MM, buy low sell high, try to arbitrage the $(p^{m2} - p^{m1})$ gap
- allowing unbounded consumptions denying CE
- therefore in CE: $p^{m1} = p^{m2} = p^m \geq 0$
Assume $p^m > 0$

- We have
  \[ p^1(x^1_h - \omega^1_h) + p^2(x^1_h - \omega^1_h) = 0 \]
- MM equilibrium allocation is identical to FM equilibrium allocation
- Irving Fisher (isomorphic to the Arrow article)
- Very important caveat
  - If $p^m = 0$, the money market is closed. No inter-temporal trades
  - This important outcome does not incur in the FM model
Uncertainty

Two states, \( s = \alpha, \beta \)

One commodity, \( l = 1 \)

Finite model, as before

See Arrow RES article: History of article

- Contingent claims
  - "AD"
  - buy and sell contracts to deliver commodity contingent on the realization of \( s \)

- \( CP: \)

\[
\max \{ \pi(\alpha)u_h(x_h(\alpha)) + \pi(\beta)u_h(x_h(\beta)) \}
\]

\[
\pi(\alpha) + \pi(\beta) = 1
\]

s.t.

\[
p(\alpha)x_h(\alpha) + p(\beta)x_h(\beta) = p(\alpha)\omega_h(\alpha) + p(\beta)\omega_h(\beta)
\]
CE for "AD" economy

\[ p(\alpha), p(\beta) \] such that

\[ \sum x_h(s) = \sum \omega_h(s) \text{ for } s = \alpha, \beta \]

where \( x_h(s) \) solves CP for \( s = \alpha, \beta \)
Arrow Securities

- Arrow money
- Buy and sell commodities on spot markets
- Buy and sell Arrow monies $b_h(s)$ for $s = \alpha, \beta$ before $s$ is realized

CP:

$$\max E_s u_h(s) = \pi(\alpha)u_h(x_h(s)) + \pi(\beta)u_h(x_h(s))$$

s.t.

$$x_h(s) = \omega_h(s) + b_h(s) \quad s = \alpha, \beta$$

and

$$p^b(\alpha)b_h(\alpha) + p^b(\beta)b_h(\beta) = 0$$

- Hidden assumption: 1 unit of $b_h(s)$ pays 1 unit of commodity in state $s$, 0 otherwise
CE:

\((p(\alpha), p(\beta); p^b(\alpha), p(\beta))\) such that

\[\sum_h x_h(s) = \sum_h \omega_h(s) \text{ for } s = \alpha, \beta\]

where \(x_h(s)\) solves CP
Results

- Every CE allocation in the AD economy can be decentralized as a CE allocation in the Arrow Securities economy.
- Every CE allocation in the AS economy in which we have $(p^b(\alpha), p^b(\beta)) >> 0$ is also an equilibrium in the AD contingent claims economy.