Economics 6130-2
Macroeconomics I, Part 2, Fall 2016
Cornell University
Practice Questions for the Final with Solutions

1. Outside Money Taxes

1.a) 5 people. Endowments $\omega = (\omega_1, \omega_2, \omega_3, \omega_4, \omega_5) = (900, 800, 700, 600, 500)$.
Find the set of equilibrium money prices $\mathscr{P}^m$ when money taxes $\tau = (\tau_1, \tau_2, \tau_3, \tau_4, \tau_5)$ are given by

1.a.i) $\tau = (2, 2, 0, -2, -2)$

Solution:

$\sum_h \tau_h = 2 + 2 + 0 - 2 - 2 = 0$, so taxes are balanced and therefore bonafide.

$$900 - 2P^m > 0 \quad \Rightarrow \quad P^m < 450$$

$$800 - 2P^m > 0 \quad \Rightarrow \quad P^m < 400$$

So $\mathscr{P}^m = [0, 400)$.

1.a.ii) $\tau = (2, 2, -1, -3, -2)$

Solution:

$\sum_h \tau_h = 2 + 2 - 1 - 3 - 2 = -2$, so taxes are *not* balanced and hence are *not* bonafide. So, $\mathscr{P}^m = \{0\}$.

1.a.iii) $\tau = (0, 0, 0, 0, 0)$

Solution:

$\sum_h \tau_h = 0 + 0 + 0 + 0 + 0 = 0$, so taxes are balanced and hence bonafide. P^m is indeterminate, and $\mathscr{P}^m = [0, \infty)$. No one net buys or net sells currency.

1.b) 3 people, two monies $B\$$ and $R\$$, with money taxes generated by

$$\tau^B = (\tau_1^B, \tau_2^B, \tau_3^B) \text{ and } \tau^R = (\tau_1^R, \tau_2^R, \tau_3^R)$$

Find the exchange rate when

1.b.i) $\tau^B = (2, 2, -2)$ and $\tau_R = (-1, -1, -1)$

Solution:

$\sum_h \tau_h^B = 2$ and $\sum_h \tau_h^R = -3$, so

$$2P^B - 3P^R = 0 \quad \Rightarrow \quad 2P^B = 3P^R$$
$$\frac{P^B}{P^R} = \frac{3}{2}$$

The exchange rate is independent of ω. To do more, however, we must specify ω. As such, it was not necessary to find the set of equilibrium money prices for this question.

1.b.ii) $\tau^B = (2, -1, -1)$ and $\tau_R = (1, 0, -1)$

Solution:

$\sum_h \tau_h^B = 0$ and $\sum_h \tau_h^R = 0$, so as

$$P^B \sum_h \tau_h^B + P^R \sum_h \tau_h^R = 0 \quad \Rightarrow \quad P^B \sum_h \tau_h^B = -P^R \sum_h \tau_h^R$$

$$\frac{P^B}{P^R} = -\frac{\sum_h \tau_h^R}{\sum_h \tau_h^B} = \frac{0}{0}$$

The exchange rate is indeterminate.

1.b.iii) $\tau^B = (1, 1, 1)$ and $\tau_R = (-1, -1, -1)$

Solution:

$\sum_h \tau_h^B = 1$ and $\sum_h \tau_h^R = -3$

$$3P^B - 3P^R = 0 \quad \Rightarrow \quad \frac{P^B}{P^R} = 1$$

1.b.iv) $\tau^B = (5, 0, 0)$ and $\tau_R = (0, 0, -10)$

Solution:

$\sum_h \tau_h^B = 5$ and $\sum_h \tau_h^R = -10$

$$5P^B - 10P^R = 0 \quad \Rightarrow \quad \frac{P^B}{P^R} = \frac{10}{5} = 2$$

2. The Diamond-Dybvig Bank

The probability of being impatient is 0.50. The utility function is:

$$u(c) = 10 - \frac{1}{(0.5)\sqrt{c}}.$$

The rate of return to the asset harvested late is 400%, i.e., $R = 5$.
The depositor's endowment is $y = 7$, which she deposits in the bank. The banking contract is (d_1, d_2), where $t = 1, 2$; it is the promised withdrawal for depositors seeking to withdraw in period t.

2.a) Graph the following in (d_1, d_2) space:
2.a.i) The resource constraint RC

Solution:

Resource Constraint: $(1-\lambda)d_2 \le (y - \lambda d_1)R$. Therefore,

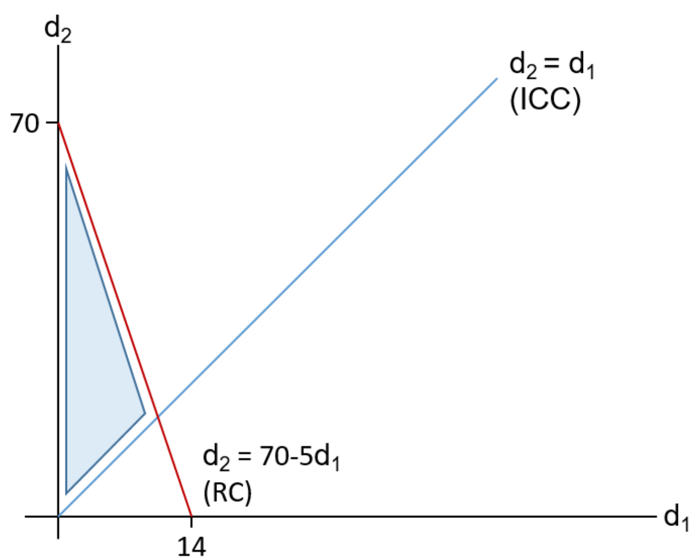
$$\frac{1}{2}d_2 \le \left(7 - \frac{1}{2}d_1\right)5$$

$$d_2 \le 70 - 5d_1 \quad \Rightarrow \quad \text{Slope of the RC} = -5$$

2.a.ii) The incentive compatibility constraint IC (or ICC)

Solution:

The ICC is $d_2 \ge d_1$. Altogether, we have



2.b) What is the depositor's *ex-ante* expected utility W as a function of c_1, consumption in period 1, and c_2, consumption in period 2? Show this in the (d_1, d_2)-space graph.

Solution:

The ex-ante expected utility of the depositor will be

$$W(c_1, c_2) = \frac{1}{2}u(c_1) + \frac{1}{2}u(c_2) = 10 - c_1^{-1/2} - c_2^{-1/2}$$

Isoquant for W: (A level set of $W = \alpha$)

$$10 - c_1^{-1/2} - c_2^{-1/2} = \alpha$$

This may be expressed as

$$c_2^{-1/2} = 10 - c_1^{-1/2} - \alpha \quad \Rightarrow \quad c_2 = (10 - c_1^{-1/2} - \alpha)^{-2}$$

$$c_2 = \frac{1}{(10 - c_1^{-1/2} - \alpha)^2}$$

This will be a concave-up isoquant with a convex preferred set; its graph will be similar to $y = \frac{1}{x}$, although it will be slightly more complicated. To draw it more precisely, we may note that along the isoquant $W = \alpha$,

$$\left(\frac{dc_2}{dc_1}\right)_{W=\alpha} = -\frac{\frac{\partial W}{\partial c_2}}{\frac{\partial W}{\partial c_2}} = -\frac{c_2^{-3/2}}{c_1^{-3/2}}$$

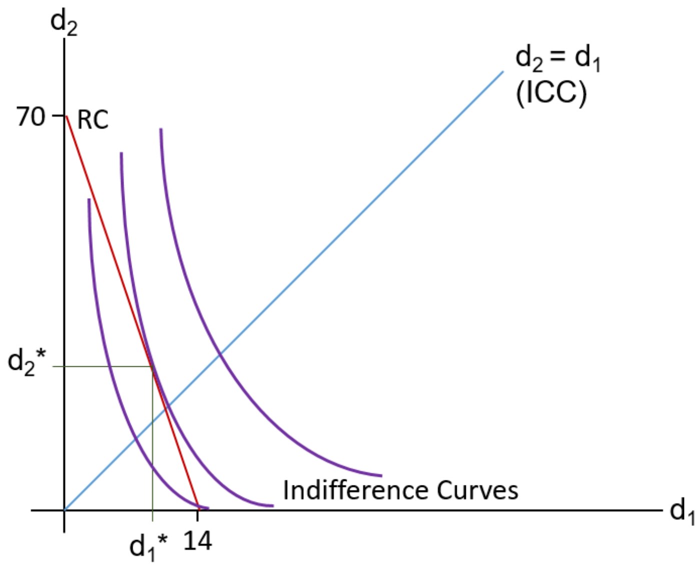
By the implicit function theorem. This leads us to

$$= -\left(\frac{c_1}{c_2}\right)^{3/2} < 0 \ \forall c_1, c_2 \in \mathbb{R}_{++}$$

So the isoquant will be downward-sloping.
For the isoquant to be tangent with the RC,

$$\left(\frac{dc_2}{dc_1}\right)_{W=\alpha,\ RC} = -5$$

This level of mathematical formalism was not explicitly required on the exam. However, it may have helped you draw the curve in the phase space.



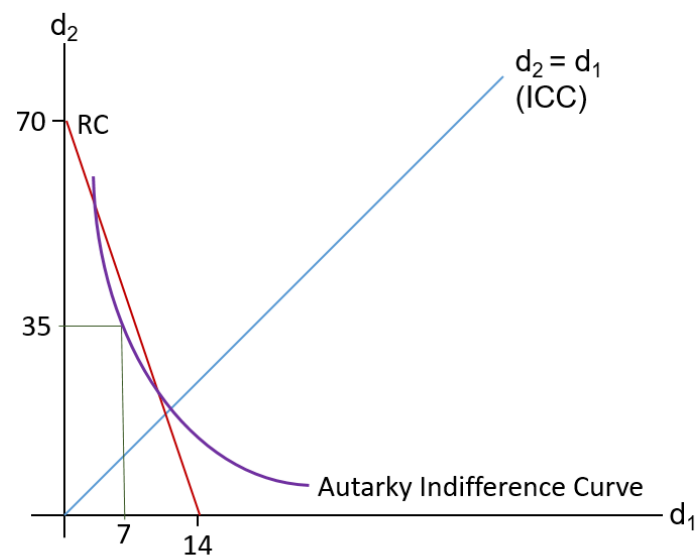
2.c) Solve for the depositor's expected utility in autarky. Show this on the graph in (d_1, d_2) space.

Solution:

In autarky,

$$W(y, yR) = \frac{1}{2}u(7) + \frac{1}{2}u(35) = \frac{1}{2}\left(10 - \frac{2}{\sqrt{7}}\right) + \frac{1}{2}\left(10 - \frac{2}{\sqrt{35}}\right)$$

$$= 10 - \frac{1}{\sqrt{7}} - \frac{1}{\sqrt{35}} \approx 9.47$$



2.d) Solve for the so-called "optimal deposit contract."

To find the optimal contract, we must solve

$$\arg\max_{d_1,d_2} \{W(d_1,d_2) = \lambda u(d_1) + (1-\lambda)u(d_2)\}$$

subject to RC and ICC. By using Lagrangian optimization, or by remembering the optimization's corollary,

$$\frac{u'(d_1)}{u'(d_2)} = R$$

$$\Rightarrow \quad \left(\frac{d_1}{d_2}\right)^{-3/2} = \left(\frac{d_2}{d_1}\right)^{3/2} = 5$$

So

$$\frac{d_2}{d_1} = 5^{2/3} \quad \Rightarrow \quad d_2 = 5^{2/3}d_1 \quad \text{and so} \quad d_1 = 5^{-2/3}d_2$$

Using the RC,

$$d_2 = 70 - 5d_1 \quad \Rightarrow \quad d_2 = 70 - 5(5^{-2/3}d_2) = 70 - 5^{1/3}d_2$$

$$d_2 + 5^{1/3}d_2 = 70 \quad \Rightarrow \quad (1+5^{1/3})d_2 = 70$$

$$d_2^* = \frac{70}{1+5^{1/3}} \approx 25.83$$

And then

$$d_1^* = 5^{-2/3}d_2^* = \frac{(5^{-2/3})(70)}{1+5^{1/3}} \approx 8.83$$

Note that $d_2^* > d_1^*$, consistent with the ICC.

2.e) What is W if there is no run? If there is a run?

Solution:

If there is no run, then the depositor simply consumes the optimal deposit contract withdrawals, d_1^*, d_2^*. Then,

$$W_{no-run} = W(d_1^*, d_2^*) = \frac{1}{2}u(d_1^*) + \frac{1}{2}u(d_2^*)$$

$$= \frac{1}{2}\left(10 - \frac{1}{\sqrt{8.83}}\right) + \frac{1}{2}\left(10 - \frac{1}{\sqrt{25.83}}\right) \approx 9.734$$

Say there is no partial suspension of convertibility in the event of a run. Then let $\hat{\lambda}$ be the fraction of depositors whom the bank can serve.

$$W_{run} = \hat{\lambda}u(d_1^*) + \hat{\lambda}u(0)$$

Since $\lim_{c \to 0} u(c) = -\infty$, we may loosely write

$$W_{run} = -\infty$$

Alternatively, suppose the bank realizes that there is a run going on after 50% of its depositors all arrive at the bank. Then, the bank switches to partial suspension of convertibility. To put this differently, the bank honors half of the depositor's demands for d_1^*, then limits withdrawals to allow for the servicing of the other half of withdrawal requests.

$$\begin{aligned} W_{run} &= \frac{1}{2}u(d_1^*) + \frac{1}{2}u\left(y - \frac{d_1^*}{2}\right) \\ &= \frac{1}{2}u(8.83) + \frac{1}{2}u\left(7 - \frac{8.83}{2}\right) \\ &= \frac{1}{2}\left[10 - \frac{2}{\sqrt{8.83}}\right] + \frac{1}{2}\left[10 - \frac{2}{\sqrt{7 - \frac{8.83}{2}}}\right] \approx 8.42 \end{aligned}$$

Note how in this solution, the run would do less damage to depositors as a whole than in the first proposed W_{run} outcome, but would still be worse than not having a run at all.

3. Overlapping Generations, I

Given

$$u_0(x_0^1) = \beta x_0^1, \quad \omega_0^1 = 1 \text{ for } t = 0$$
$$u_t(x_t^t, x_t^{t+1}) = x_t^t + \beta x_t^{t+1}, \quad (\omega_t^t, \omega_t^{t+1}) = (1, 1) \text{ for } t = 1, 2, ...$$

Where $\beta \in \mathbb{R}$ is a scalar. Let p^t be the present price of the commodity delivered at date t, i.e. $p^1 = 1$. Let $p^{m,t}$ be the present price of money at date t (in terms of the period 1 commodity). In each of the following three cases,

i. Solve for the reflected, translated offer curve (OC).

ii. Graph the OC.

iii. Provide the phase diagram and the full dynamic analysis, including the steady states and their stability.

iv. Calculate $\mathscr{P}^m$, the set of equilibrium money prices.

[Hint: Calculus is not the best tool for analyzing the linear model]

Case A: $\beta = \frac{1}{2}$, $m_0^1 = 1$, $m_t^s = 0$ otherwise.

Case B: $\beta = 2$, $m_0^1 = 2$, $m_t^s = 0$ otherwise.

Case C: $\beta = 2$, $m_0^1 = 0$, $m_1^1 = m_1^2 = 1$, $m_t^s = 0$ otherwise.

Solution:

i. For all of the cases, the solution to i) will be the same. It is possible to solve the problem in multiple ways; all should yield the same result. The fastest derivation may be done by drawing the indifference curve through the endowment, thereby finding the MRS. The Offer Curve is then the union of the indifference curve and the parts of the axes above the IC. To find the reflected offer curve, we then need only to re-write the original offer curve in terms of the excess demand of the old generations.

An indifference curve of the utility function will be a level set such that

$$u_t(x_t^t, x_t^{t+1}) = \bar{u} = x_t^t + \beta x_t^{t+1}$$

To draw the curve *through* the endowment, we may then set $\bar{u} = u(\omega_t^t, \omega_t^{t+1})$ to get

$$x_t^t + \beta x_t^{t+1} = \omega_t^t + \beta \omega_t^{t+1} \quad \Rightarrow \quad \beta(x_t^{t+1} - \omega_t^{t+1}) = (\omega_t^t - x_t^t) \quad \Rightarrow \quad \beta z^{t+1} = z^t$$

Where $z^{t+1} = x_t^{t+1} - \omega_t^{t+1}$ and $z^t = (\omega_t^t - x_t^t)$. The indifference curve through the origin is thus represented in terms of (z^{t+1}, z^t) as

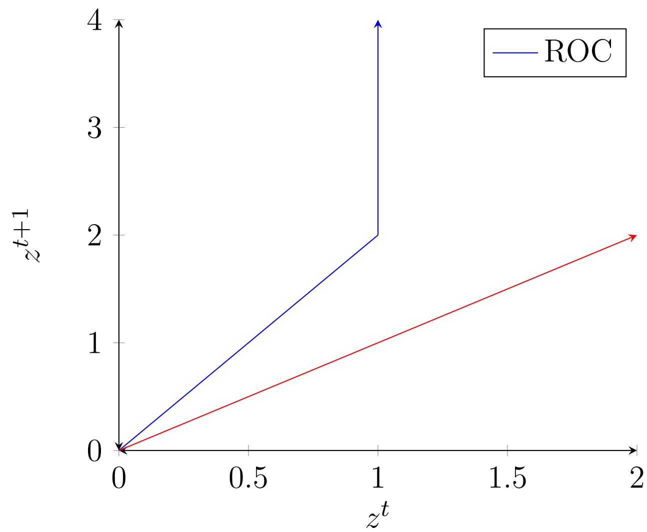
$$z^{t+1} = \frac{1}{\beta} z^t$$

The offer curve of the old generation is the the union of the indifference curve with the parts of the axes above the IC. This is equivalent to noting that $z^t \leq \omega_t^t = 1$, as the old generation cannot consume more than the young generation is able to offer them. Thus, due to the constraint of physical resources, a vertical line will interrupt the reflected offer curve where $z^t = 1$.

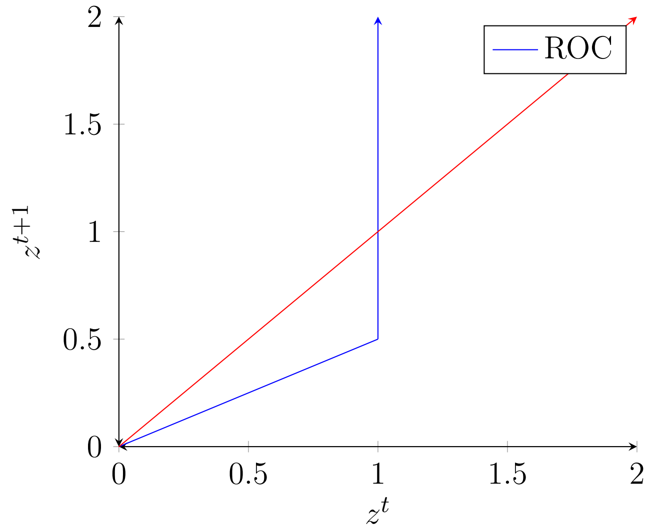
Solving the problem via Lagrangian optimization or by setting the MRS equal to the price ratio will yield exactly the same answer, although the calculation will be somewhat longer.

To make one final comment on the notation, we may observe that $z^t = -s_t^t = z_{t-1}^t$, while $z^{t+1} = z_t^{t+1}$. The excess demand of the old generation is equal to the excess supply of the young generation for each time period t, as the market will clear.

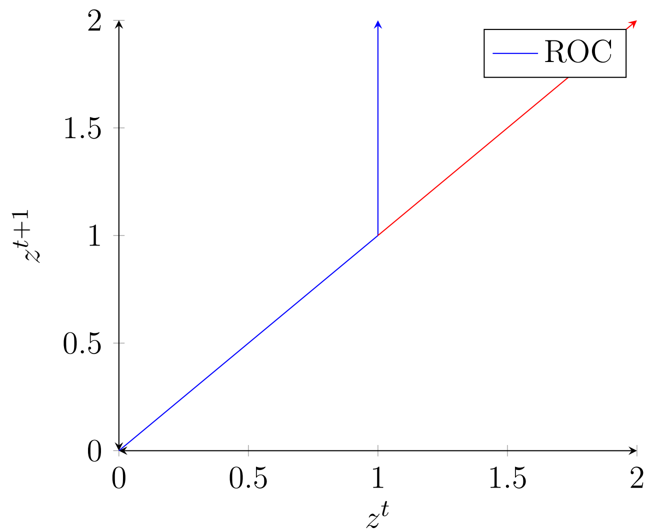
ii. For Case A, since $\beta = \frac{1}{2}$ the reflected offer curve is $z^{t+1} = 2z^t$.



In contrast, for Cases B and C, the reflected offer curve is $z^{t+1} = \frac{1}{2}z^t$, as $\beta = 2$.



While this wasn't a case, we may also note that if $\beta = 1$ such that $z^{t+1} = z^t$, the graph will be as follows:



iii. and iv. We must either assert or prove a no-arbitrage profits condition such that $p^{m,t} = p^{m,t+1} = p^m \geq 0$.

In Case A, as $MRS = \frac{1}{1/2} = 2$ such that $r = 1 > 0$, we must be in the Ricardo case. There exists one stationary equilibrium, such that $z^t = z^{t+1} = 0$, the autarky state. For $z^1 \in (0, \infty)$, the money bubble bursts in finite time. For $z^1 = 0$ such that $z^t = z^{t+1} = 0$, the money bubble never forms. $p^m = 0$ is Pareto optimal, and $\mathscr{P}^m = \{0\}$.

In Case B, $MRS = \frac{1}{2}$ so that $r < 0$, implying we are in the Samuelson case. From there, we may solve to find the monetary steady state as $z^t = z^{t+1} = \frac{1}{2}$. Any $z^t > 1$ is infeasible, as each old generation can buy only up to one unit of consumption from their progeny. Thus, the initial old generation will purchase goods in excess of their endowment only what is available on the market:

$$z^1 = p^m m_0^1 \leq \omega_1^1 \;\Rightarrow\; 2p^m \leq 1 \Rightarrow p^m \leq \frac{1}{2}$$

Therefore $\bar{p}^m = \frac{1}{2}$ and the set of equilibrium money prices is $\mathscr{P}^m = [0, \frac{1}{2}]$. This $\bar{p}^m = \frac{1}{2}$ is Pareto optimal, but it is *not* stationary. $p^m \in (0, \frac{1}{2})$ is not optimal, but it is Pareto-superior to the non-monetary autarky; if the economy starts in autarky ($\bar{p} = 0$), the money bubble will again never form. If $p^m \in (0, \frac{1}{2}]$, the money will inflate, price levels will rise, and the bubble will fade away asymptotically.

In Case C, the problem is quite similar to that described in Case B. Since $m_0^1 = 0$ but $m_1^1 = m_1^2 = 1$, we may treat the Ur Son generation Mr. 1 as we treated the Ur Father (the Mr. 0 generation) in the problem above. Any $z^t > 1$ is once more infeasible, so when the Ur Son generation is old,

$$z^1 = p^m(m_1^1 + m_1^2) \leq \omega_2^2$$

$$2p^m \leq 1 \quad \Rightarrow \quad p^m \leq \frac{1}{2}$$

The equilibrium set of money prices is therefore again $\mathscr{P}^m = [0, \frac{1}{2}]$. As the offer curve is the same for Case B, the possible dynamics of the system remain the same as in the prior case.

4. Overlapping Generations, II

$$u_0(x_0^1) = 2\log x_0^1, \quad \omega_0^1 = 100 \text{ for } t = 0$$
$$u_t(x_t^t, x_t^{t+1}) = \log x_t^t + 2\log x_t^{t+1}, \quad (\omega_t^t, \omega_t^{t+1}) = (100, 100) \text{ for } t = 1, 2, ...$$

Let p^t be the present price of the commodity, $p^1 = 1$. $p^{m,t}$ is the present price of money in terms of the period 1 commodity. Additionally, let $m_0^1 = 1$ and $m_t^s = 0$ otherwise.

a) Calculate the marginal rate of commodity substitution (MRS) at the endowment $(100, 100)$.

b) What is the rate of interest at the endowment?

c) Is the economy Samuelson, or is it Ricardo? Why?

d) Derive the reflected, translated offer curve (OC).

e) Graph the OC.

f) Conduct the complete dynamic analysis in the phase diagram, including:

   i. Calculation of steady states.
   ii. Calculation of the set of money prices $\mathscr{P}^m$.
   iii. Stability analysis.
   iv. Inflation.
   v. Deflation.
   vi. Welfare.

Solutions:

a) We may find the MRS by finding the partial derivatives of utility with respect to consumption in each time period, although there are other methods that may work just as well.

$$MRS = -\frac{dx_t^{t+1}}{dx_t^t} = \frac{\frac{\partial u}{\partial x_t^t}}{\frac{\partial u}{\partial x_t^{t+1}}} = \frac{\frac{1}{x_t^t}}{\frac{2}{x_t^{t+1}}} = \frac{x_t^{t+1}}{2x_t^t}$$

Which at the endowment $(100, 100)$ is

$$MRS(\omega_t^t, \omega_t^{t+1}) = \frac{100}{200} = \frac{1}{2}$$

b) As $MRS = \frac{p^t}{p^{t+1}} = R$, it holds that $R = \frac{1}{2}$ such that $r = R - 1 = -\frac{1}{2}$.

c) Since $r < 0$, the slope of the offer curve of Agent t is $\frac{1}{2} < 1$. We are thus in the Samuelson case.

d) Rather than doing the entire optimization problem from scratch, we may recall from b) that the price ratio between periods will be $\frac{p^t}{p^{t+1}} = \frac{1}{2}$. Substituting back into the budget constraint,

$$p^t x_t^t + p^{t+1} x^{t+1} = p^t \omega_t^t + p^{t+1} \omega^{t+1} \quad \Rightarrow \quad p^{t+1}(x_{t+1}^t - \omega_{t+1}^t) = p^t(\omega_t^t - x_t^t)$$

$$p^{t+1} z^{t+1} = p^t z^t$$

$$z^{t+1} = \frac{p^t}{p^{t+1}} z^t \ \Rightarrow z^{t+1} = \frac{x_t^{t+1}}{2x_t^t} z^t = \frac{z^{t+1} + \omega_t^{t+1}}{2\omega_t^t - 2z_t^t} z^t$$

$$\frac{z^{t+1}}{z^{t+1} + \omega_t^{t+1}} = \frac{z^t}{2\omega_t^t - 2z_t^t} \quad \Rightarrow \quad \frac{z^{t+1} + \omega_t^{t+1}}{z^{t+1}} = \frac{2\omega_t^t - 2z_t^t}{z^t} \quad \Rightarrow \quad 1 + \frac{\omega_t^{t+1}}{z^{t+1}} = \frac{2\omega_t^t}{z^t} - 2$$

$$\frac{\omega_t^{t+1}}{z^{t+1}} = \frac{2\omega_t^t}{z^t} - 3 \quad \Rightarrow \quad \frac{1}{z^{t+1}} = \frac{2\omega_t^t - 3z^t}{\omega_t^{t+1} z^t}$$

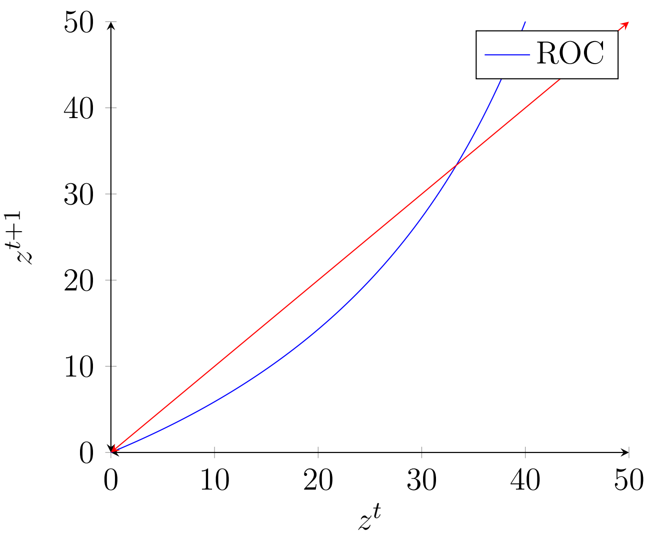
$$z^{t+1} = \frac{\omega_t^{t+1} z^t}{2\omega_t^t - 3z^t}$$

The (reflected) offer curve is therefore

$$z^{t+1} = \frac{100 z^t}{200 - 3z^t}$$

Students who memorized the format of the answer as $z^{t+1} = \frac{BCz^t}{AD-(C+D)z^t}$ were given full credit, although students who derived it algebraically received extra credit.

e) The graph of the ROC is as follows:



f.i) At a steady state, $z^{t+1} = z^t$. One steady state will be $z^t = 0$. The other will satisfy

$$z = \frac{100z}{200 - 3z} \quad \Rightarrow \quad 200 - 3z = 100 \;\Rightarrow\; z = \frac{100}{3}$$

$z^t = 0$ will be a non-monetary, non-Pareto optimal steady-state. $z^t = \frac{100}{3}$, however, will be a Pareto Optimal fixed point.

f.ii) We may either show or assert a no-arbitrage profits condition, such that $p^{m,t} = p^{m,t+1} = p^m \geq 0\ \forall t$. The set of equilibrium money prices is then

$$\mathscr{P}^m = \left[0, \frac{100}{3}\right]$$

f.iii) $z^t = 0$ is a stable no-monetary (autarky) steady state; trajectories that begin in the neighborhood of $z^0 \in \left(0, \frac{100}{3}\right)$ will tend asymptotically to it. $z^t = \frac{100}{3}$ is in contract unstable.

f.iv) If $0 < p^m < \frac{100}{3}$, the money bubble will fade away through hyperinflation.

f.v) If $p^m > \frac{100}{3}$, then the bubble will burst in finite time through hyperdeflation. Note that neither hyperinflation nor hyperdeflation will occur if individuals avoid "disequilibrium" paths.

f.vi) $p^m = \frac{100}{3}$ is Pareto Optimal, while $p^m = 0$ is not. $p^m \in \left(0, \frac{100}{3}\right)$ is not a steady state and is not Pareto Optimal, but it is Pareto Superior to $p^m = 0$.