1. Outside Money Taxes

1.a) 5 people. Endowments $\omega = (\omega_1, \omega_2, \omega_3, \omega_4, \omega_5) = (900, 800, 700, 600, 500)$. Find the set of equilibrium money prices $P_m$ when money taxes $\tau = (\tau_1, \tau_2, \tau_3, \tau_4, \tau_5)$ are given by

1.a.i) $\tau = (2, 2, 0, -2, -2)$
1.a.ii) $\tau = (2, 2, -1, -3, -2)$
1.a.iii) $\tau = (0, 0, 0, 0, 0)$

1.b) 3 people, two monies $B$ and $R$, with money taxes generated by $\tau^B = (\tau^B_1, \tau^B_2, \tau^B_3)$ and $\tau^R = (\tau^R_1, \tau^R_2, \tau^R_3)$

Find the sets of money prices and the exchange rate when

1.b.i) $\tau^B = (2, 2, -2)$ and $\tau^R = (-1, -1, -1)$
1.b.ii) $\tau^B = (2, -1, -1)$ and $\tau^R = (1, 0, -1)$
1.b.iii) $\tau^B = (1, 1, 1)$ and $\tau^R = (-1, -1, -1)$
1.b.iv) $\tau^B = (5, 0, 0)$ and $\tau^R = (0, 0, -10)$
2. The Diamond-Dybvig Bank

The probability of being impatient is 0.50. The utility function is:

\[ u(c) = 10 - \frac{1}{(0.5)\sqrt{c}}. \]

The rate of return to the asset harvested late is 400\%, i.e., \( R = 5 \).

The depositor’s endowment is \( y = 7 \), which she deposits in the bank. The banking contract is \((d_1, d_2)\), where \( t = 1, 2 \); it is the promised withdrawal for depositors seeking to withdraw in period \( t \).

2.a) Graph the following in \((d_1, d_2)\) space:
   2.a.i) The resource constraint RC
   2.a.ii) The incentive compatibility constraint IC

2.b) What is the depositor’s ex-ante expected utility \( W \) as a function of \( c_1 \), consumption in period 1, and \( c_2 \), consumption in period 2? Show this in the \((d_1, d_2)\)-space graph.

2.c) Solve for the depositor’s expected utility in autarky. Show this on the graph in \((d_1, d_2)\) space.

2.d) Solve for the so-called “optimal deposit contract.”

2.e) What is \( W \) if there is no run? If there is a run?
3. Overlapping Generations, I

Given
\[ u_0(x^1_0) = \beta x^1_0, \quad \omega^1_0 = 1 \text{ for } t = 0 \]
\[ u_t(x^t_t, x^{t+1}_t) = x^t_t + \beta x^{t+1}_t, \quad (\omega^t_t, \omega^{t+1}_t) = (1, 1) \text{ for } t = 1, 2, \ldots \]

Where \( \beta \in \mathbb{R} \) is a scalar. Let \( p^t \) be the present price of the commodity delivered at date \( t \), i.e. \( p^1 = 1 \). Let \( p^{m,t} \) be the present price of money at date \( t \) (in terms of the period 1 commodity). In each of the following three cases,

i. Solve for the reflected, translated offer curve (OC).

ii. Graph the OC.

iii. Provide the phase diagram and the full dynamic analysis, including the steady states and their stability.

iv. Calculate \( \mathcal{P}^m \), the set of equilibrium money prices.

[Hint: Calculus is not the best tool for analyzing the linear model]

Case A: \( \beta = \frac{1}{2}, \quad m^1_0 = 1, \quad m^s_i = 0 \text{ otherwise.} \)

Case B: \( \beta = 2, \quad m^1_0 = 2, \quad m^s_i = 0 \text{ otherwise.} \)

Case C: \( \beta = 2, \quad m^1_0 = 0, \quad m^1_1 = m^2_1 = 1, \quad m^s_i = 0 \text{ otherwise.} \)
4. Overlapping Generations, II

\[ u_0(x_0^1) = 2 \log x_0^1, \quad \omega_0^1 = 100 \text{ for } t = 0 \]

\[ u_t(x_t^t, x_{t+1}^t) = \log x_t^t + 2 \log x_{t+1}^t, \quad (\omega_t^t, \omega_{t+1}^t) = (100, 100) \text{ for } t = 1, 2, \ldots \]

Let \( p^t \) be the present price of the commodity, \( p^1 = 1 \). \( p^{m,t} \) is the present price of money in terms of the period 1 commodity.

a) Calculate the marginal rate of commodity substitution (MRS) at the endowment \((100, 100)\).

b) What is the rate of interest at the endowment?

c) Is the economy Samuelson, or is it Ricardo? Why?

d) Derive the reflected, translated offer curve (OC).

e) Graph the OC.

f) Conduct the complete dynamic analysis in the phase diagram, including:

i. Calculation of steady states.

ii. Calculation of the set of money prices \( \mathcal{P}^m \).

iii. Stability analysis.

iv. Inflation.

v. Deflation.

vi. Welfare.