Bank Runs: The Pre-Deposit Game

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- One cannot understand bank runs or the optimal contract without the full *pre-deposit* game
- Peck and Shell (2003): A *sunspot-driven* run can be an equilibrium in the *pre-deposit* game for sufficiently small run probability.
- We show *how* sunspot-driven run risk affects the optimal contract depending on the parameters.

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- Types are uncorrelated (so we have aggregate uncertainty.): p

The Model: Technology

- Storage:

$$\begin{array}{lll} t=0 & t=1 & t=2 \\ -1 & 1 & \\ & -1 & 1 \end{array}$$

The Model: Technology

- Storage:

$t = 0$	$t = 1$	$t = 2$
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- More Productive

$t = 0$	$t = 1$	$t = 2$
-1	0	R

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- Aggregate uncertainty

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- Smaller c is conservative; larger c is fragile

Post-Deposit Game: c^{early}

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- Let c^{early} be the value of c such that the above inequality holds as an equality.

Post-Deposit Game: c^{wait}

- A patient depositor chooses late withdrawal when he expects the other depositor, if patient, to also choose late withdrawal. (ICC)

$$pv[(2y - c)R] + (1 - p)v(yR) \geq p[v(c) + v(2y - c)]/2 + (1 - p)v(c).$$

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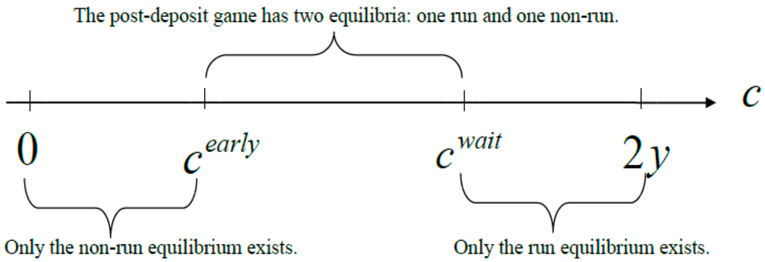
- Let c^{wait} be the value of c such that the above inequality holds as an equality.

Post-Deposit Game: "usual" values of the parameters

- $c^{early} < c^{wait}$ if and only if

$$b < \min\{2, 1 + \ln 2 / \ln R\}$$



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- We call these values of b and R "usual" since the set of DSIC contracts (i.e, $[0, c^{wait}]$) is a strict subset of BIC contracts (i.e, $[0, c^{early}]$).

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- The interval $(c^{early}, c^{wait}]$ is the region of c for which the patient depositors' withdrawal decisions exhibit *strategic complementarity*.

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- The values of b and R are “unusual” when the set of DSIC contracts is the same as the set of BIC contracts.
- According to the Revelation Principle, when we search for the optimal contract we only have to focus on the BIC contracts.
- Hence, for the “unusual” parameters, the optimal contract must be DSIC and the bank runs are not relevant.

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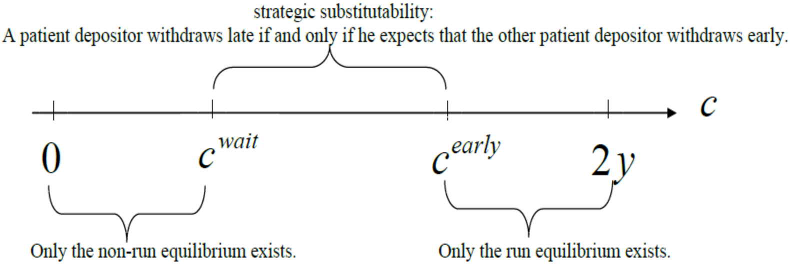
- $(c^{wait}, c^{early}]$ is the region of c for which the patient depositors' withdrawal decisions exhibit *strategic substitutability.*

Figure 8. Equilibrium in the Post-Deposit Game



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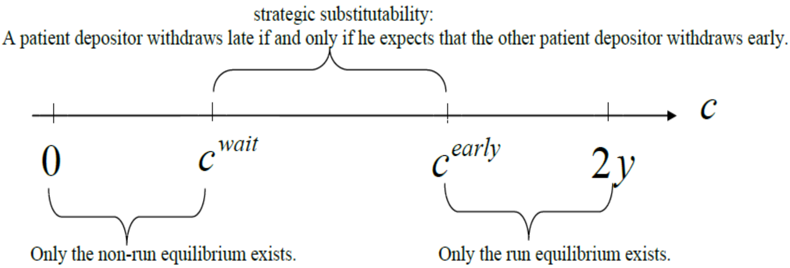

- For the optimal contract, the only relevant region is $[0, c^{wait}]$ (i.e., BIC contracts).

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  - $\widehat{c} \leq c^{early}$ (Case 1)
  - $\widehat{c} \in (c^{early}, c^{wait}]$ (Case 2)
  - $\widehat{c} > c^{wait}$ (Case 3)

Impulse parameter A and the 3 cases

- $\widehat{c}$ is the c in $[0, 2y]$ that maximizes

$$\widehat{W}(c) = \{p^2[u(c) + u(2y - c)] + 2p(1-p)[u(c) + v((2y - c)R)] \\ + 2(1-p)^2 v(yR)\}.$$

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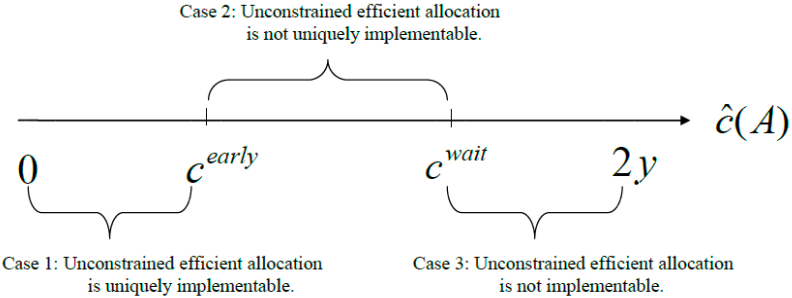
- $\widehat{c}(A)$ is an increasing function of A.

Parameter A and the 3 Cases

- Neither c^{early} nor c^{wait} depends on A

Figure 2. Three Cases



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$$b = 1.01; p = 0.5; y = 3; R = 1.5$$

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- $A^{early} = 6.217686$ and $A^{wait} = 10.27799$.

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- $A^{early} = 6.217686$ and $A^{wait} = 10.27799$.
- If $A \leq A^{early}$, we are in Case 1; If $A^{early} < A \leq A^{wait}$, we are in Case 2; If $A > A^{wait}$, we are in Case 3.

The Optimal Contract: Case 1

- Case 1: The *unconstrained efficient allocation* is DSIC, i.e., $\widehat{c} \leq c^{early}$.

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- It is straightforward to see that the optimal contract for the *pre-deposit* game supports the *unconstrained efficient allocation*

$$c^*(s) = \widehat{c}.$$

and that the optimal contract doesn't tolerate runs.

The Optimal Contract: Case 2

- Case 2: The *unconstrained efficient allocation* is BIC but not DSIC, i.e., $c^{early} < \widehat{c} \leq c^{wait}$.

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- The optimal contract $c^*(s)$ satisfies: (1) if s is larger than the threshold probability s_0, the optimal contract is run-proof and $c^*(s) = c^{early}$. (2) if s is smaller than s_0, the optimal contract $c^*(s)$ tolerates runs and it is a strictly decreasing function of s.

The Optimal Contract: Case 2

- Using the same parameters as the previous example. Let $A = 8$. (We have seen that we are in Case 2 if $6.217686 < A \leq 10.27799$.)

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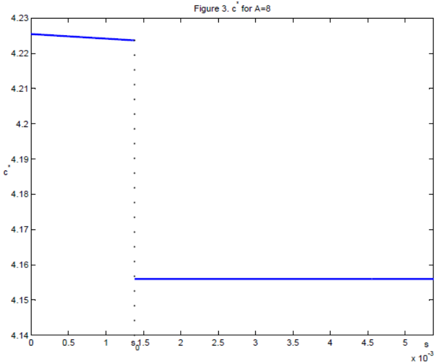
- c^* switches to the best run-proof contract (i.e. c^{early}) when $s > s_0 = 1.382358 \times 10^{-3}$.



The Optimal Contract: Case 3

- Case 3: The *unconstrained efficient allocation* is not BIC, i.e., $c^{wait} < \widehat{c}$.

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- The optimal contract $c^*(s)$ satisfies: (1) If s is larger than the threshold probability s_1, we have $c^*(s) = c^{early}$ and the optimal contract is run-proof. (2) If s is smaller than s_1, the optimal contract $c^*(s)$ tolerates runs and it is a weakly decreasing function of s. Furthermore, we have $c^*(s) = c^{wait}$ for at least part of the run tolerating range of s.

The Optimal Contract: Case 3

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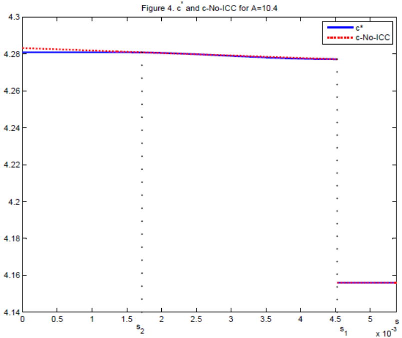
- c^* switches to the best run-proof (i.e. c^{early}) when $s > 4.524181 \times 10^{-3}$.
- ICC becomes non-binding when $s \geq s_2 = 1.719643 \times 10^{-3}$.



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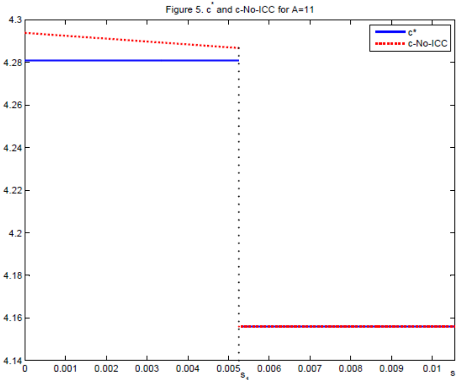
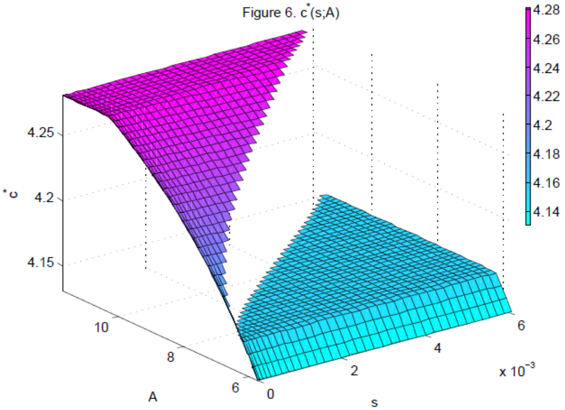
- c^* switches to the best run-proof (i.e. c^{early}) when $s > s_1 = 5.281242 \times 10^{-3}$.



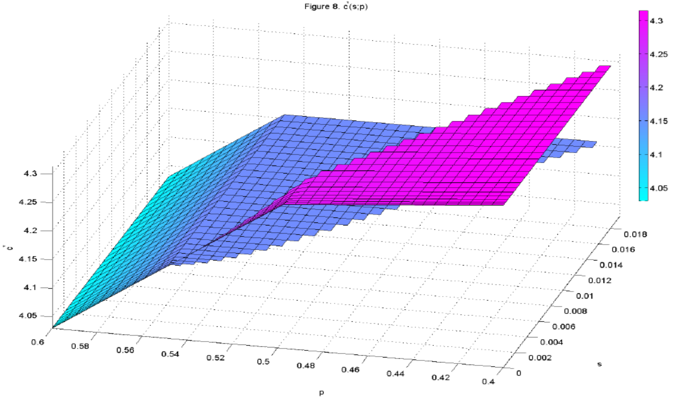
The Optimal Contract

- c^* versus s and A



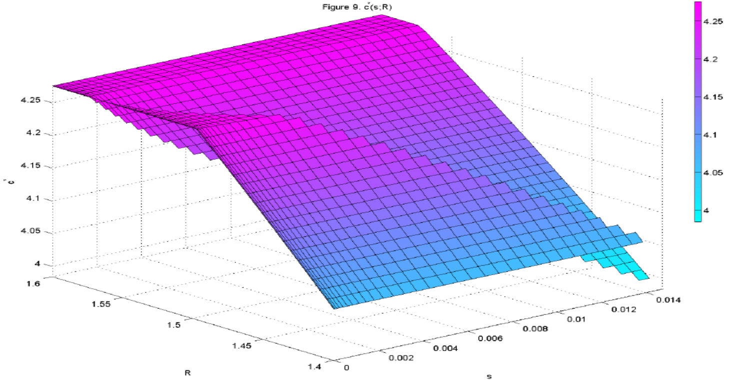
Probability of Impatience: p

- $b = 1.01$, $A = 10$, $y = 3$, $R = 1.5$. If $p \geq 0.548823$, the optimal contract does not tolerate runs, $c^*(s) = \widehat{c}$. If $p \in [0.497423, 0.548823)$, then c^* is strictly decreasing in s until it levels off to $c^{early} = 4.155955$. If $p < 0.497423$, then the ICC binds when s is small.



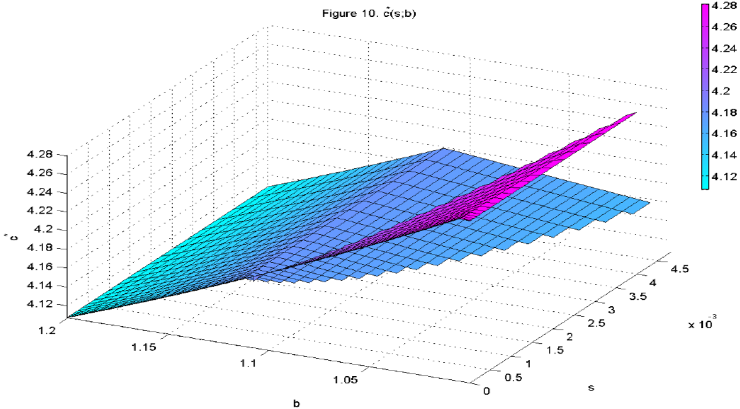
Return R

- $b = 1.01$, $A = 10$, $y = 3$, $p = 0.5$. If $R \geq 1.572948$, the optimal contract does not tolerate runs, $c^*(s) = \widehat{c}$. If $R \in [1.497374, 1.572948)$, $c^*(s)$ is strictly decreasing in s until it levels off to c^{early}. $c^{early}(R)$ is increasing in R. If $R < 1.497374$, then the ICC binds when s is small.



Risk-aversion b

- $A = 10$, $y = 3$, $p = 0.5$, $R = 1.5$. If $b \geq 1.112528$, the optimal contract does not tolerate runs, $c^*(s) = \widehat{c}$. $\widehat{c}$ depends on b. If $b \in [1.00524, 1.112528)$, then $c^*(s)$ is strictly decreasing in s until it levels off to c^{early}. If $b < 1.00524$, then the ICC binds when s is small.



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- In Case 2, the optimal contract adjusts continuously and becomes strictly more conservative as the run probabilities increases.
  - The optimal allocation is never a mere randomization over the *unconstrained efficient allocation* and the corresponding run allocation from the *post-deposit* game. Hence this is also a contribution to the sunspots literature: another case in which SSE allocations are not mere randomizations over certainty allocations.

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- In Case 3, the ICC binds for small run-probabilities, which forces the contract to be more conservative than it would have been without the ICC. Hence, for Case 3, the optimal contract does not change with s until the ICC no longer binds.

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  - For small s, the optimal allocation is a randomization over the *constrained efficient allocation* and the corresponding run allocation from the *post-deposit* game.